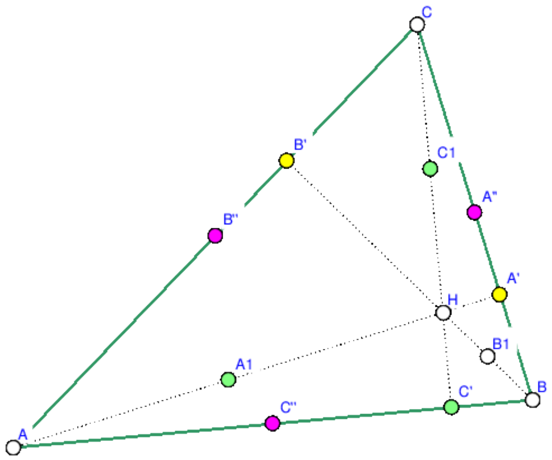
OK Geometry Report :

Author:: -

Source: K:\2013_Koper\NinePoint3.pro

Comment:

TASK



1

NINE POINT CIRCLE

Given is a triangle △ABC, let H be its orthocentre.

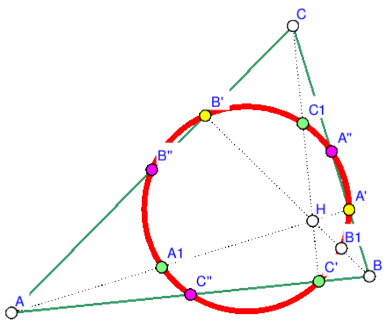
Let A', B', C' be the bases of the heights of ABC.

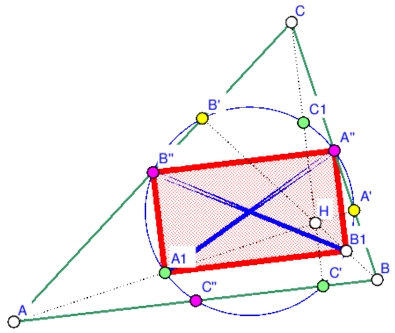
Let A",B",C" be the midpoints of the sides of △ABC.

Finally, let A1,B1,C1 be the midpoints of the segments AH, BH, CH

Then the points A',B',C', A",B",C", A1, B1,C1 lay on a circle called nine-point circle of △ABC. Furthermore, the segments A1A", B1B", C1C" are diameters of the nine-point circle.

WORKOUT

	2 Cocircular points - B'A'C'A"B"C"B1C1A1	

<table>
<tr>
<td>
</td>
<td>3.0
Rectangle -
A"B"B1A1</td>
<td>Why A1B1 and B"A" are parellel? (See details.)</td>
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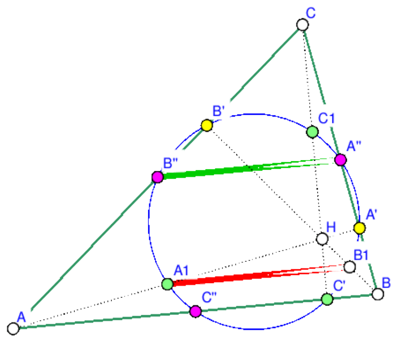
<tr>
<td>
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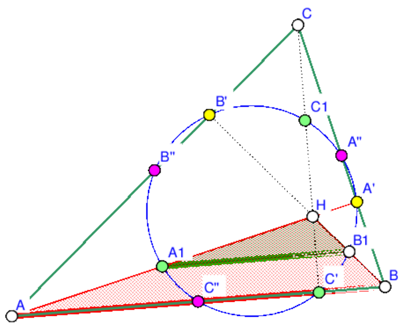
<td>3.1.0
Parallel segments:
A1B1 || B"A"</td>
<td>Why are A1B" and B1A" parallel ? (See details)</td>
</tr>
<tr>
<td>
</td>
<td>3.1.1
AB || A1B1</td>
<td>A1 is the midpoint of AH.

B1 is the midpoint of BH.

By Thales theorem △ABH and △A1 B1H are similar and

AB || A1B1</td>
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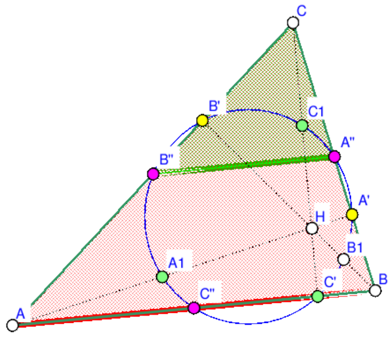
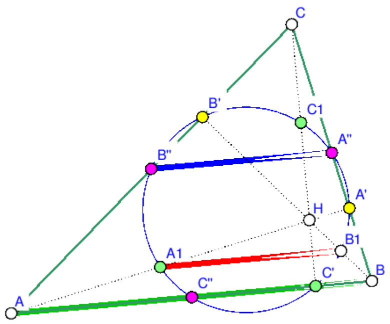
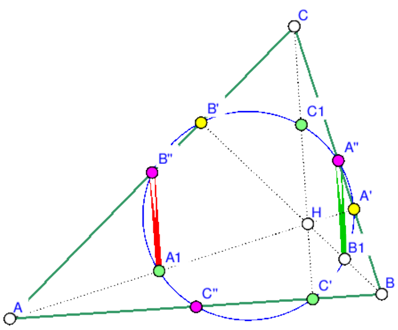
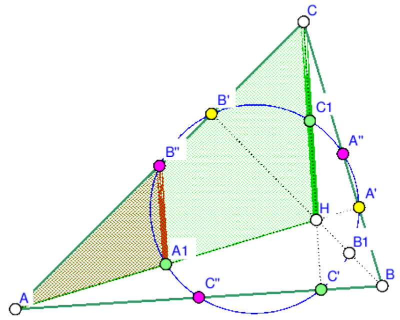
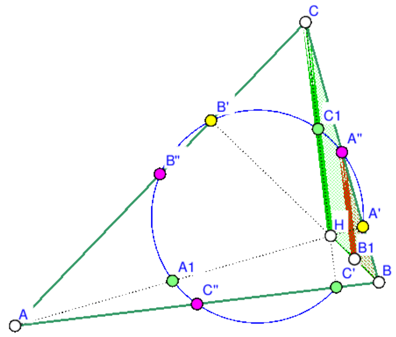
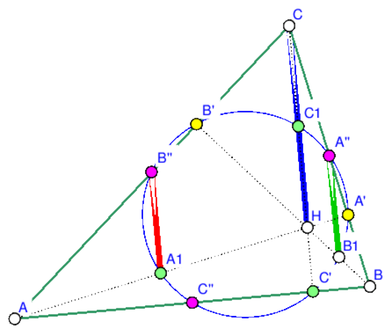
	3.1.2 B"A" || AB	
	3.1.3 A1B1 || B"A"	AB || A1B1 by #3.1.1 B"A" || AB by #3.1.2 By transitivity of || thus A1B1 || B"A"
	3.2.0 Parallel lines - A1B" || B1A"	

	3.2.1 A1B" || HC	A1 is the midpoint of AH (by def.) B" is the midpoint of AC (by def.) By Thales theorem △AA1B" and △AHC are similar, and A1B" || HC .
	3.2.2 B1A" || HC	B1 is the midpoint of BH (by hyp.) A" is the midpoint of BC (by hyp.) By Thales theorem △BB1A" and △BHC are similar, and B1A" || HC .
	3.2.3 A1B" || B1A"	A1B' || HC by #3.2.1 HC || B1A" by #3.2.2 By transitivity of || A1B" || B1A"

<table>
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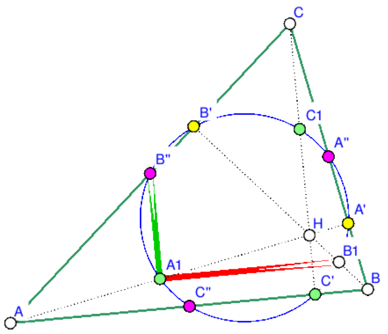

</td>
<td>3.3.0
Orthogonal segments -
A1B1 ⊥ A1B"</td>
<td></td>
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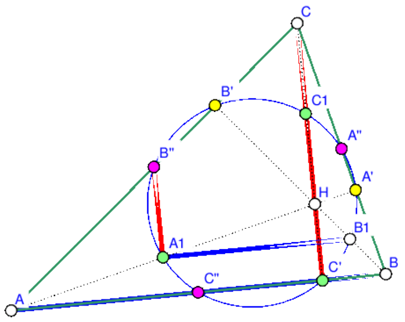
<td>


</td>
<td>3.3.1
A1B" ⊥ A1B1</td>
<td>A1B" || CC' by #3.2.1

A1B1 || AB by #3.1.1

AB ⊥ CC' since CC' is the altitude to AB.

Thus

A1B" ⊥ A1B1</td>
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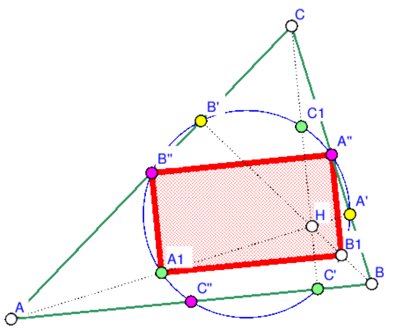
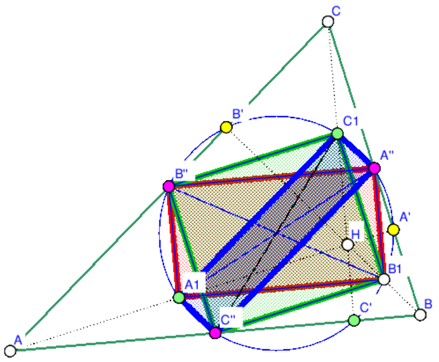
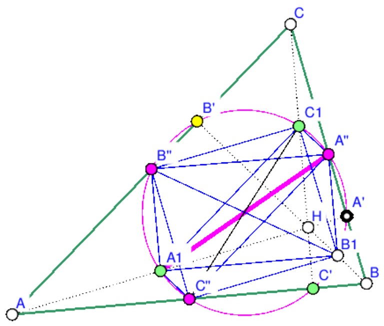
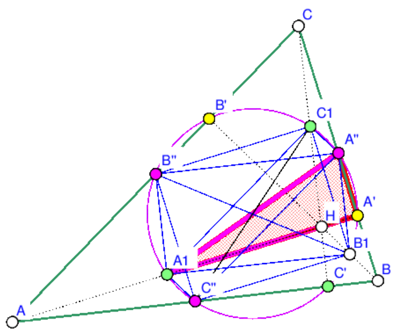
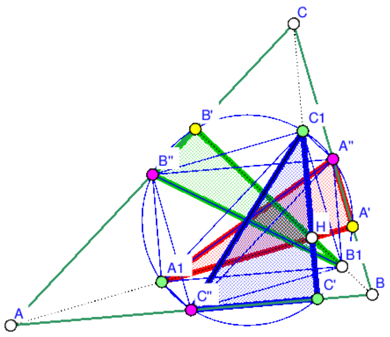
<tr>
<td>


</td>
<td>3.4
Rectangle -
A"B"B1A1</td>
<td>By 3.1 and 3.2 A"B"B1A1 is a parallelogram..

By 3.3 is a right-agled parallelogram, thus, a rectangle.</td>
</tr>
</table>

	4 rectangles - A"B"B1A1, B"C"B1C1, C"A"C1A1	A1C"A"C1 and B1C1B"C" are parallelograms by the same arguments as stated in 3.
	5.0 A' lays on the circle which has A1A"A'A' as diameter	B' and C' lay on circle with diameter B"B1 and C1C" by the same arguments as explained in 5.
	5.1 Right angle #A"A'A1	$\triangle$A1A'A" is right angled at A'. since AA" is the altitude to BC. By Thales theorem A' lays on the semicircle above A1A".

	6 A',B',C' lay on circle through A"B"C"	

Notes