# OK Geometry Basic <br> Part I <br> Reference for OK Geometry Basic (v.17.2) <br> Zlatan Magajna 

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## 1 What is OK Geometry?

OK Geometry is an aid for analysing geometric constructions. Given a geometric construction, OK Geometry detects patterns in the construction as well as certain properties of the construction. OK Geometry may be also a help for organising the found information.

OK Geometry was written in first place as a tool for students. Students often find difficult to cope with geometry exercises like construction tasks or tasks that require building up a proof. Solving such tasks requires an appropriate organisation of the properties of the configuration. A necessary step in solving many geometry tasks is thus the awareness of the properties of a configuration. OK Geometry may help students to overcome this step.

Teachers may find this program useful for presenting formal proofs to students (in electronic or written format). Teachers may use the program for generating exercises, but this is indeed not the aim of the program.

Recreational mathematicians interested in classical geometry may find this program as a useful tool for analysing various geometric configurations, in particular those related to triangle geometry. OK Geometry contains a huge database of characteristic points, lines, circles, and conics that are used in analysing objects with regard to a given triangle.

You may think of OK Geometry as a pair of geometric spectacles. Using sophisticated numerical and non-numerical methods OK Geometry detects potential properties of dynamic geometric constructions (e.g. that certain points are collinear or that certain triangles have the same area). Students should be aware that one thing is seeing (observing) a property, another thing is to know that a property is true and still another thing is to have an explanation why a property is necessarily true. OK Geometry helps the student in the first step. OK Geometry will not prove that something is true, it is even possible (though quite improbable) that it will make an 'observational error'. OK Geometry helps to observe a configuration, and this is an important step towards knowing why something in the configuration is true. OK Geometry also helps to relate a given object of a configuration to other objects (Is there anything in this configuration related to a given angle?). OK Geometry may also help the user to organise properties, but it will not select which properties are important, interesting or even novel.

To summarise: OK Geometry helps the user to analyse geometric configurations and to reason about its properties, but it is up to the user to decide which properties are relevant for his purpose and, eventually, to use them in building up a proof.

### 1.1 The model behind OK Geometry

Planar geometry is about configurations of geometric objects like points, lines, circles etc. Such objects and configurations can be considered form a naive perspective - however, formally, in geometry such objects are abstract entities and the properties of configurations are deduced from an assumed set of axioms, from which a system of definitions, theorems is build. Geometric reasoning requires that geometric objects are in some way represented, no matter whether one considers them a naive fashion or in a formal way. The representations of geometric objects are usually based on a model, so that not only the objects but also the properties among them can be represented in some way. A model thus helps to visualise and understand properties, and can be used to informally check the validity of claims
and to hypothesise new properties. In contrast with the 'paper and pencil' way of representing geometric objects, which is based on the static model, and in contrast with the dynamic geometry software, which is based on the dynamic model, OK Geometry uses the stochastic dynamic model.

Before we briefly explain the three models we need to clarify an aspect of geometric constructions. Consider, for example, an equilateral triangle (Figure 1). To construct a general equilateral triangle we may start from two of its vertices, say $A$ and $B$, which can be positioned arbitrarily on the plane. The third vertex of the triangle, say $C$, has to positioned so that its distance from $A$ and from $B$ is the same as the distance between $A$ and $B$. We say that $A$ and $B$ are non-constructed or free objects, while $C$ is a constructed object. The line segments $A B, B C$, and $C A$ are, obviously, also constructed objects since they are determined by previously set objects.


Figure 1
In the static model all represented objects, whether free or constructed, are fixed. For example, the plane is represented as a sheet of paper, points by drawn dots on it, etc. The drawn objects are static.

In the dynamic model the plane is represented, for example, by a computer display and the objects by dots or sets of dots on it. The free (non-constructed) points can be dragged. The constructed objects cannot be dragged but they change in accordance with the construction and the position of free points.

The stochastic dynamic model is similar to the dynamic model. However, the free (non-constructed) objects are represented by several copies of randomly positioned objects. The constructed objects are represented by several objects constructed from copies of other objects in accordance with the construction. Only one copy of each object is visible, and assumes a reasonably random position (as well as its copies).

| The static model. All |  |
| :--- | :--- |
| represented objects are fixed. | The dynamic model. The points <br> A and B can be dragged. All <br> other objects change <br> accordingly. |
| The stochastic dynamic model. A <br> and B can be dragged. There are <br> several randomly positioned <br> copies of each object - only one <br> of them is visible. |  |

Figure 2
The three models are illustrated in Figure 2 for the case of equilateral triangle with given points A and B . The difference between the models comes to light when a property is checked. In the static model an observed property (based on a measurement) can be easily a matter of chance. The same is true in the dynamic model; however, by - using dragging - other examples of construction can be done and checked for the property. In the stochastic dynamic model a property is simultaneously checked on all random copies of the object, thus there is no need for dragging when inspecting a property.

### 1.2 Exact and perturbated view. The blue dot

The main job ob OK Geometry is observing dynamic constructions. In doing this OK Geometry always uses stochastic dynamic representation of objects, so that it considers several copies of perturbed points around each given free point, though it shows only one (perturbed) construction. However, sometimes may prefer to visualise the construction based on the very given free points. For this reason OK Geometry can visualise both, the exact and the perturbed view.

The blue dot on the left upper corner of the displayed construction serves as an indicator as well as a switch between the exact and the perturbed visualisation. When in exact mode, the dot appears like

- 

, while in perturbed mode the blue dot contains some random red pixels . The size of the blue dot shows the size of the perturbation area around free points. A click on the blue dot changes the modes from one to the other.

Internally, all observations are made in the perturbed mode. On the other hand, all exports of constructions are made in the exact mode, irrespective of the status of the blue dot.

Note that some constructions cannot be displayed in exact mode.
The power of OK Geometry is, by no means, in the perturbed representation of objects. Sometimes viewing objects in this mode is not desired. For example, points and the whole construction changes
slightly from time to time, in this mode we cannot draw horizontal or vertical lines. On the other hand, in the exact mode, we can draw perfect horizontal and vertical lines (OK Geometry gives a help to do this). There is also a command in Sketch editor for visually aligning points.

A right click on the blue dot open allows the configuration of some often used parameters related to the display of the configuration (e.g. the way of filling areas, font of point labels). In particular, you can also set the level of the displayed help for the construction commands.

### 1.3 Easy, Basic, and Plus mode of OK Geometry

OK Geometry works in three modes. The Easy mode is designed for occasional users - for it enables to analyse geometric constructions made by dynamic geometry software (like GeoGebra, Cabri, Cabri Express, Sketchometry, Cinderella, C.a.R. (Compass and Ruler), Sketchometry or JGEX). The Easy mode is also designed for solving exercises in deductive reasoning in planar geometry: such exercises can be prepared with OK Geometry (but not in the Easy mode). To access the Basic mode click Configure / Working mode /Basic in the menu (Figure 3). This document considers only the Basic mode of OK Geometry, which is also part of the Plus mode. (See Help for Easy mode in the Help menu to learn about the Easy mode.)


Figure 3
When the title of the main program window is just OK Geometry the program works in Basic mode. Here is a concise list of tasks that OK Geometry can do in the Basic mode:

- import constructions from some dynamic geometry software (DGS), currently for Cabri, Cabri II, Cabri Express, GeoGebra, Cinderella, CaR (Compass and Ruler), Sketchometry, JGEX (Java Geometry Expert);
- make new constructions or edit the imported ones;
- modify constructions in order to achieve certain goals (e.g. automatically move a point in order to obtain a maximal area of a shape in a construction);
- spot patterns and properties in geometric constructions;
- investigate properties related to a specific object in a construction;
- select relevant properties for further elaboration;
- organise the selected information (e.g. to build up a proof);
- present properties of a configuration (or a proof);
- make a report that can be read with common word processors or browsers.

This document describes how to use the OK Geometry in Basic mode.

## 2 Some terminology conventions

In the text we use the term (geometric) object or entity to denote any basic geometric object such as point, line, circle, arc, (line) segment, polyline (i.e. a concatenated sequence of line segments). By extension, this list sometimes includes also real numbers (related to geometric quantities) and real valued variables (parameters).

By (geometric, dynamic) construction we understand a set of free basic objects (points or real numbers) and a sequence of objects that are obtained from free or previously constructed objects using 'allowed' construction methods. The 'allowed' construction methods should be reducible to basic Euclidean construction steps.

By (geometric) configuration we understand a set of geometric objects positioned in Euclidean plane, usually satisfying specified relations.

We shall use the term icon for a pictorial representation of a geometric property. An icon consists of a configuration with some emphasized elements, together with a related verbal comment and the underlying dynamic construction. An icon may be related to several scenes. Scenes are just visualisations of an icon in which chosen objects are made more or less emphasized and more or less visible.

Several icons are organised in a project. A project may depict interesting properties of a geometric configuration or, for example, the steps of a proof of a theorem. The basic (starting) icon of the project is named Task.


Figure 4
The two central concepts we shall often use are: geometric properties and instances of properties. We shall use the term (geometric) property for general properties, e.g. collinearity of points, perpendicularity of lines. By instance (of a property) or relation we mean a property applied to specific objects in a configuration or construction, e.g. the fact that the points $A, B, C$ in a configuration are collinear is a relation, thus an instance of the property collinearity.

Figure 4 (left) displays a geometric construction (right pane of the left screenshot), its properties and instances of properties (left pane of the left screenshot). A construction can - together with some comment - turned into an icon. Figure 4 (right) displays a project consisting of several icons.

OK Geometry is often used in a very simple fashion (Figure 5), just to study a construction:

1. A dynamic construction is made in Sketch editor, which is a module of OK Geometry, or imported from a dynamic geometry software.
2. The geometric properties of the construction are analysed and studied.

## construction

## Figure 5

OK Geometry allows also a more complex schema of work (Figure 6). The work, i.e. project, may consist of several commented constructions. A project may be about a theorem and its proof, about properties of a geometric shape, about a family of geometric objects, about a group of related theorems, etc.


Figure 6

## 3 The interface of OK Geometry

On the top of the window there is a menu line with sets of commands (File, Configure, Commands, and Help). Below the menu line there is the main menu bar with buttons for the basic commands. Still below the main menu bar is the information pane, which consists of several sections. The sections can be minimised, expanded or fully expanded - to change their state click or right-click the button (with up/down oriented triangle) on the left of the title of the section. Finally, on the right there is a visualisation pane. The visualisation pane displays a geometric construction (Figure 7) or the icons of a project. The details of the interface in either case differ somewhat, but are rather self evident.

There are several ways to access help as you work in OK Geometry. Some commands with help information are located under the rightmost menu item. Additional help information for entries is often available as self-appearing balloons or by right-clicking on object or entry. During the Sketch operation there is also a Help section for editing commands, the section can be turned on or off.


Figure 7

## 4 Introductory examples

### 4.1 First example. Detecting properties

We illustrate how to detect geometric properties with Ok Geometry on a very simple situation: Given is a triangle $A B C$ and a point on each of its sides: point $D$ on $B C$, point $E$ on $C A$, point $F$ on $A B$. We shall analyse this configuration.

Although it is possible to draw constructions in OK Geometry, in this introductory example we start with construction files made with dynamic geometry software Cabri, Cabri II, Cabri Express, GeoGebra, C.a.R. (Compass and Ruler), Sketchometry, Cinderella or JGEX (see Figure 8). You can find the related files (Example1.fig or Example1.ggb, Example1.jc, Example1.zir or Example1.cdy) among the provided examples.


Figure 8
Open OK Geometry in Basic mode and follow these steps:

- Click the button Task in the main menu bar. A form for selecting the file with construction (in Cabri, Cabri Express, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), Sketchometry, Sketchometry, JGEX or Ok Geometry) appears. After you choose your file, e.g. Example1.fig, the construction should appear on the right pane.
- Click the button Observe. A list of detected properties of the considered construction appears in the left pane in section Properties (see Figure 9).
- Click on a property you are interested in, e.g. concurrent circles. The property 'expands' and instances of the property appear. If you click on an instance of a property the property is visualised on the right pane. Many of the listed properties are trivial (e.g. the collinearity of points $A, B, F$. However, you may find properties you were not aware of, e.g. that the some circles through labelled points pass through a common point (Figure 10) - a fact that should not surprise those who have heard of Miquel theorem.


Figure 9


Figure 10

### 4.2 Second example. Querying objects in a configuration

Often OK Geometry detects several properties and several instances of properties. If we interested only in properties related to some object in the configuration, we can use the query commands. Here is a simple example (see Example4.p).


Figure 11
Given are circles $k(M, A)$ and $k(N, B)$ that intersect in points $P$ and $Q$ (Figure 11). Let $C$ be the intersection, other than P , of the line BP and circle $k(\mathrm{M}, \mathrm{A})$. Let D be the intersection, other than P , of the line AP and circle $k(\mathrm{~N}, \mathrm{~B})$. OK Geometry lists on advanced treatment more than a hundred of properties of this simple configuration, among them 38 about angle congruence. Suppose we want to find information about the angle between lines AC and BD. We start from a construction
in some dynamic geometry software (Example4.ggb, Example4.jc, Example4.txt, Example4.zir, Example4.cdy) and proceed as in the first example.

1. Using the command Task we import the desired dynamic construction.
2. Obtain a list of observed properties using the command Observe (Figure 12).


Figure 12
3. On the right taskbar a series of query buttons appears. Click QueryAngle, then pick the segment AC (click somewhere in on the segment) and the segment BD.
4. Select the query level (on top of display). On the level 3 or 4 OK Geometry list just one relation: the angle between lines AC and BD is supplementary to the angle CQB (which gives rise to a cyclic quadrilateral, see Figure 13). On level 1 or 2 we come to know of 5 angles in the configuration of the same size as the considered one.


## Figure 13

The query commands include functions that may be of great help in solving construction tasks.

### 4.3 Third example. Working on detected properties

Consider the following task: The diagonals of a trapezium split the trapezium into four triangles. Prove that two of the resulting triangles have the same area.

Also in this example we shall rely on some program for dynamic geometry. Using, for example, Cabri, Cabri Express, Cinderella , C.a.R. (Compass and Ruler), JGEX or GeoGebra, let us construct a trapezium (see Figure 14). Note that the vertices should be labelled each with a single letter followed by digits or apostrophes. We also write the text of the task as a text in the program of dynamic geometry. We save the construction in file Example2.fig (or Example2.ggb or Example2.cdy). You can use the files in the

provided examples.
Figure 14
Now open OK Geometry and follow these steps:


Figure 15

Click the button Task in the main menu bar. A form for selecting the file with construction (in Cabri, Cabri Express, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), Sketchometry, JGEX, or Ok Geometry) appears. Find your file, e.g. Example2.fig, select it and confirm by clicking the OK button. The construction of the quadrilateral should appear on the right pane (Figure 15). Since we intend to work on properties of this configuration, we declare this imported construction as the basic icon, the Task. To do this click on the leftmost icon above the visualisation pane and declare the construction as 1 Task (Figure 15).
Click the button Observe on the main menu bar to produce a list of observed properties of the considered construction.
Among the listed properties click on those you find related to your problem, in our case the areas of triangles. The property 'expands' and three specific properties appear. Click (but, for now, do not double click) on any of them, e.g. $A B C^{\sim} \sim A B D$, to visualise it. If you want to visualise a specific object in the relation, e.g. the triangle $A B C$, click on the name of the object above the drawing pane. (See Figure 16.)


Figure 16
The observed relations that could possibly be relevant for further consideration should be declared as icons (this can be done at any stage of process). For example: triangles AED and BEC have the same area, and triangles $A B C$ and $A B D$ have the same areas as well. This is clearly related to the fact that in a trapezium the bases $A B$ and $C D$ are parallel. In order to add the relation to the current construction click on the red ' + ' icon above the visualisation pane (Figure 15). The construction together with the emphasised relation is displayed in the Sketch mode, since you may eventually want to make some changes to the displayed configuration. Usually the necessary changes can be done with simple button-ready commands that appear. See Figure

17 for the most important among them. When a relation involving angles is displayed it is usually necessary to use the anchor button to select among the angle between two intersecting lines. Other changes may be done as well, for example changing the colours and styles of the displayed objects, hiding objects, deleting objects, bleaching objects. Note that the objects that illustrate the relation are emphasised as relation objects: it is possible to change their colour and style, but not the width of lines.
It is not a good idea to drag free points or to re-label the points of the configuration since the icon will be (probably) used in relation to the basic task (Task icon). Anyway, in principle any changes are allowed and the obtained construction declared as an icon.
When the presentation of the relation is as desired, declare it as a New icon (or overwrite a previous one) using the $\rightarrow \square$ button. A form appears (Figure 18).


Figure 17
Fill the form, e.g. write a comment why the relation is true (you can write and edit comments also later). It is possible to attribute various statuses to the paragraphs: definition/description (D), claim/hypothesis (H), argumentation (A), comment, and formula. Figure 18 shows a commented form for the relation that triangles $A B D$ and $A B C$ have the same area). Then Click OK in the form if you want to use the property in further work


Figure 18
Once you have selected the relevant instances of properties you may proceed to organising them in order to document them or even to build up a proof of a claim. In the main task bar click the Project button (Figure 19). On the left pane you will see the previously selected instances of properties, while on the right pane there will be icons representing the relations. You can move the icons around so that they are positioned in a logical order. If you want to add a change a comment to the relation, simply double click the description on the left or the icon.


Figure 19

If you want to change the order of the relations either: a) use the + and - key on the keyboard, or b) drag the relations on the left pane, or c) right-click on the icon to be renumbered and choose the appropriate command. To change the attributes of the icons (size of icons, size of labels, emphasis and bleaching effect, size of angle notation) use the sliders.

The icons of a project can be presented all at once as (Figure 19) or one-by-one (Figure 20). To present the icons one-by-one click on the button. Then use the presentation buttons to move between steps/icons. Note: As explained latter, the proof can be organised in a linear fashion or in a tree-like fashion. In the latter case it is possible to set the level of details in the proof and navigate in the steps of the proof.


Figure 20
Finally, a report can be made from the selected properties and the added comments. Note that before producing reports it is necessary to install a Ghostscript interpreter ${ }^{1}$. The report - written in HTM format - can be easily read and edited with common HTM viewers and text processing programs. Functionally, the report can be either a memo of the found relations or a classical two column proof of a statement (as built by the user). To make a report click Report on the main tool bar and follow the instructions. In the form that will appear you will be asked for some data (Figure 21) the name of the report file and then to fill a form like the form below:

[^0]

Figure 21

## Here is how a report might look like:

## A trapezium task

1 Task


Here is a trapezium ABCD . The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.

## Comment:

## 2 parallel lines

Claim $1 \mathrm{AB} \| \mathrm{CD}$
Argument 1 By definition of trapezium, since ABCD is a trapezium.


## 3 same areas

Claim $1 \operatorname{area}(\triangle \mathrm{ABC})=\operatorname{area}(\triangle \mathrm{ABD})$
Argument 1 Triangles $\triangle A B C$ and $\triangle A B D$ share the same basis AB and have equal heights because of \#2.

## 4 areas of triangles

Claim $1 \operatorname{area}(\triangle \mathrm{ADE})=\operatorname{area}(\triangle \mathrm{BCE})$
Argument 1 From the equal areas, see \#3, of $\triangle A B C$ and $\triangle A B D$ we subtract the common area of $\triangle A B E$. The
 remaining parts, $\triangle \mathrm{AED}$ and $\triangle \mathrm{BCE}$, have the same area.

## 5 Understanding properties

In order to give an idea how OK Geometry treats geometric properties we give a very simple example.
Suppose that one makes two similar constructions in a dynamic geometry system, consisting of just three points. We explain the construction, how OK Geometry interprets them.

|  | Construction 1 | Construction 2 |
| :---: | :---: | :---: |
| Construction in a dynamic geometry system e.g. Geogebra | free point $A(0,0)$ <br> free point $B(1,0)$ <br> free point $C(2,0)$ | free point $A(0,0)$ <br> free point $C(2,0)$ <br> constructed point $B=$ midpoint of $A$ and $C$ |
| Visual presentation in a dynamic geometry system <br> Visual presentation in exact visualisation of OK Geometry | $0^{A} \quad 0^{B} \quad 0^{C}$ | $0^{A} \quad 0^{B} \quad 0^{C}$ |
| Interpretation of the construction in OK Geometry | random point A (several random variants, only one displayed) random point B (several random variants, only one displayed) <br> random point C (several random variants, only one displayed) | random point A (several random variants, only one displayed) random point C (several random variants, only one displayed) <br> $B$ is midpoint of $A$ and $C$ (in each variant of point $A, C$, and $B$ ) |
| Visual presentation in perturbed visualisation of OK Geometry |  | $0_{0} \quad 0^{B} \quad 0^{C}$ <br> $A$ and $C$ often attain new random position, B is always the midpoint of $A$ and $C$. |
| Observation of OK Geometry | Nothing is observed. | $B$ is the midpoint of $A$ and $C$. |

Thus the constructions in OK Geometry are often apparently (slightly) shaken; however, the geometric properties that result from the construction are preserved.

In the analysis of constructions OK Geometry considers

- all displayed labelled points (unlabelled points are ignored);
- all displayed lines, circles, and similar objects;
- all lines (displayed or not) passing through any two displayed labelled points;
- all circles (displayed or not) passing through any three displayed labelled points;
- all conics (displayed or not) passing through any five displayed labelled points;
- $\quad$ specific objects with vertices on displayed labelled points (eg. squares, displayed or not)

The displayed points may be labelled or not, however, only labelled points are taken into account in the analysis. Remember:

- Point labels consist of one or more letters eventually followed by one or more digits, apostrophes. Here are some examples of allowed labels: A, B, b, P, $c^{\prime}, D^{\prime \prime}, E 13, t 2^{\prime}, A b$. Labels like E13 or Abc are sometimes displayed as $\mathrm{E}_{13}$ and ( Abc ). For example, the vertices of the triangle $\Delta \mathrm{E} 4$ ( Ad ) P are: E4, Ad , P. Labels are case sensitive. Lowercase $x, y, z$ cannot be used as labels. Different points cannot share the same label.
- A reasonable number of labelled points to consider in a construction is 4-12. The number of spotted relations, in general, rapidly increases with the number of considered points in a construction. An excessive number of considered points may result in an unmanageable collection of properties or to an unclear visual representation of the properties. During the analysis it is possible to exclude some labelled points from being considered (see section 8.1 on how to exclude points from the analysis and 7.5 on modifying labels).
- OK Geometry considers only acute and right angles between lines. Properties are thus often stated in the form that something holds for some angles or their supplementary angles. The reader has to find out where to take into consideration the angles and where the supplementary angles. In the Figure 22 (right) OK Geometry, for example, notes -referring to $\angle \mathrm{C}$ and $\angle \mathrm{D}$ - the ratio 1:2 between the two 'displayed angles or their supplementary angles'. In fact, the ratio 1:2 is between the angle indicated by green colour and the supplement of the angle indicated by red colour.
- OK Geometry does not warn of trivial cases of congruency of angles (angles related to pairs of parallel lines, angles formed by two crossing lines), for any pair of parallel lines would produce an immense number of congruent angles. Thus, when considering the spotted properties one should check if there are parallel lines and be aware of the congruency of angles defined by parallel lines and lines that cross them. In the trapezium (Figure 22 left), for example, OK Geometry does not list the fact that the angles $\angle \mathrm{DEA}$ and $\angle \mathrm{BEC}$ or $\angle \mathrm{DCA}$ and $\angle \mathrm{CAB}$ are congruent.


Figure 22

The decision which points to label in a geometric construction is very important. In general, try to label all points that you find to be potentially relevant to your investigation. In most common cases it is quite straightforward which points to label. However, especially when solving harder problems, a good choice of points is also a matter of good intuition.

## 6 Working in OK Geometry

When using OK Geometry the workflow consists of three steps. At any time it is possible to move back to a previous step. The steps are:

Step 1. Obtain an appropriate geometric construction. A construction can be obtained from a dynamic geometry software or using the Sketch editor, a module of OK geometry. The construction should follow the lines described in Section 5. For more detailed information about importing constructions see Sections 7.1. Detailed instructions for the embedded Sketch editor are in the Sketch editor manual.

Step 2. Analyse the properties of a construction. Analysing a construction is very simple. Just press the button Observe in the main menu bar and inspect the properties (some simple adjustments are sometimes needed). See the help manual for the Simple mode for a short introduction in this topic. When complex or specific properties are considered, advanced operations described in the table below should be considered.

Step 3 (optional). Document the observations, eventually organise them into a proof. Any listed observation can be added to the construction (with the ' + ' button), edited, commented and declared as icons with the $\rightarrow \square$ button (see Figure 16). The iconised observations may be later edited (using the Project command in the main menu bar) and saved as a HTML file with the Report command in the main menu bar. For more details see Sections 10 and 12.

OK Geometry can be used in a simple way: one just imports a geometric construction and uses OK Geometry to obtain a list of properties. It is also easy to produce a report, if one decides to do so. However, an advanced user will feel the need to control the process, e.g. to edit a construction or to decide what geometric properties should be checked. For this purpose special operations are available. The table below summarizes common steps in simple workflow and the related specific operations.

| Purpose | Simple operation | Specific operations |
| :--- | :--- | :--- |
| Import a construction from <br> Cabri, Cabri Express, GeoGebra, <br> Cinderella, JGEX, Sketchometry <br> or OK Geometry. <br> Edit a construction or make a <br> new one in OK Geometry. | Press the button Task in the <br> Main menu bar and import <br> the construction. | Overviewing dynamic constructions <br> made in various dynamic systems <br> Importing constructions from Cabri, <br> Cabri Express, Geometry, GeoGebra, |
| Cinderella, , C.a.R. (Compass and <br> Ruler), $\frac{\text { JGEX, Sketchometry }}{}$ <br> Making a construction in OK |  |  |
| Geometry |  |  |


|  |  | Editing geometric constructions <br> Labelling points <br> Writing text related to a task <br> Saving and reading constructions and projects <br> Recovering the work |
| :---: | :---: | :---: |
| Obtain a list of properties of the construction. | Press the button Observe in the main menu bar. | Setting the points to be considered in the analysis <br> Setting the level of analysis <br> Taking into account the symmetries of geometric constructions <br> Perturbation radius <br> Detecting properties <br> Interpreting the list of detected properties <br> Visualising a relation |
| Select and comment relevant properties. | Click the relation to visualise it. <br> Edit, comment and iconise selected relations | Searching specific relations <br> Adding displayed objects to the construction <br> Comment a relation or icon <br> Images and formulae in comments |
| Organise the selected properties into a document. | Press the Project button to visualise the selected properties. Organise the properties by dragging their icons. Double click them for editing.Drag the relations in the left pane to change the order of the relations. Right click the properties or icons for additional operations. | Organising the icons <br> Commenting icons <br> Editing icon's construction <br> Setting a scene <br> Menaging scenes <br> Conforming icons <br> Presenting the project <br> Saving the project |
| Produce a HTM file of the report. | Press the Report button and follow the instructions. | Producing reports |
| Additional themes |  | Configuring OK Geometry |

## 7 Geometric constructions

### 7.1 Importing constructions

Currently, it is possible to import dynamic constructions from

- Cabri Geometry (Cabri Geometry, Cabri Geometry II, Cabri Geometry II Plus),
- Cabri Express,
- GeoGebra,
- Cinderella,
- C.a.R. (Compass and Ruler),
- Sketchometry,
- JGEX,
- Sketchometry,
- OK Geometry.

Cabri, Cabri Express, GeoGebra, and Cinderella files can be stored in native formats. In order do be read by OK Geometry the C.a.R. constructions should be written in non-compressed native format (.zir), the Sketchometry files should be exported as .jc files, and JGEX construction need to be exportd in text form (.txt).

To import a construction click the Task button in the main menu bar and select the file you want to import. You can access to this command also from menu File/Open construction or from menu Commands/Task.

OK Geometry imports from other formats objects and properties relevant for geometric analysis.
In short, there is just one thing to care about when making a construction to be imported in OK Geometry: construct polygons (triangles) by line segments rather than as a single entity (since intersection points of polygons with other objects may cause trouble).

Here are some detailed information on importing constructions from any of the above mentioned dynamic geometry software:

Labelling points. Try to label points as required by OK Geometry (See Section 5). In any case, the point labels can be easily modified in OK Geometry. The labels of objects other than points are ignored.
Other objects. OK Geometry imports most types of geometric objects: points, lines and line segments, vectors, axes, polygons, circles, arcs, loci, geometric measurements, numeric values, calculations, all transformations, all constructions, and also macros. There are no restrictions for labels of objects other than points since these labels are ignored. Note that:
Vectors and axes are imported in OK Geometry but are not displayed.
Conics are imported in a limited way.
Rays are imported as lines.
Polygons are imported, but intersection of polygons with other objects may cause problems for some file formats. The safe way of intersecting a polygon with another object is to construct side base-lines or polygon sides as segments and intersect them with other objects. Thus,
instead of intersecting a triangle with a line it is better to separately intersect the sides' baselines with the given line.

Properties of objects. The visibility and fixedness of objects is, in general, preserved. Use the command Configure/General options/Import attributes of objects to set whether colour and other attributes of object should be (reasonably) preserved when importing objects.

Text. All text entities are accumulated and imported in OK Geometry as task description.
Magnification. The construction should be made in a reasonable magnification (e.g. between 1:1 to $1: 10$ ). An excesses in this respect may lead to errors in analysis.

### 7.2 Overviewing files from various dynamic systems

The command File/Inspect files allow the visual preview of various formats of files in a directory. The previewed constructions allow some editing and exporting. A previewed construction can be accepted, otherwise the overviewing does not interfere with current work.


Figure 23

### 7.3 Creating and editing constructions in OK Geometry

You can edit or make new geometric constructions in the Sketch editor, i.e. in Sketch mode of OK Geometry. You can access the editor by pressing the Sketch button in the main menu bar or via the menu Commands/Sketch. To exit from the Sketch editor press any button in the main menu bar.

The Sketch editor is similar to common editors of dynamic geometry. However, due to specifics of the program, there are some simplifications (e.g. no animation), some specific emphasis, and some specific commands (e.g. commands for complex and implicit geometric constructions). The idea of Ok Geometry

Sketch editor is to enable to 'construct' a configuration even if one does not know how to construct it as Euclidean construction. OK Geometry may then help in analysing the configuration, perhaps discover properties one was not aware of and find ways to do a proper construction and prove its correctness.

Please, refer took Sketch Editor Reference for information how to use the editor. There you will also find several examples.

### 7.4 Exporting figures

The displayed construction can be exported at any moment as a picture in various formats with the command File/Export image in the main menu or in context menu obtained by a right clicking on the construction pane. Note that the exported image is always (internally) done in exact visualisation mode. The image can be exported in three ways:

- Via clipboard for fast copy/pasting of image into some other application.
- Writing the image of the construction in some raster format. You can choose among the commonly used formats (PNG, GIF, JPG, BMP, WMF, TIFF). Such formats are compatible with most commonly used applications. Make sure that the resulting image quality is appropriate (12000 pts or more).
- Writing the image of the construction in a vectorised format (PS (PostScript), PDF, EMF, EPSI). These formats give high precision images in applications that can process them.

Note. Exporting requires a prior installation of a Ghostscript interpreter ${ }^{2}$. Exporting to EMF (Extended Metafile Format) currently also requires a prior installation of Inkscape.

Exporting strategy. Here is an efficient way for exporting images of constructions.

1. Obtain a desired display of the construction. Note that only the indicated square area in the construction pane will be exported. Then call the export command.
2. Decide about the desired format/mode of export. If your option is the clipboard or a file in raster format, set the Image quality to the desired value (e.g. 1200). In case of vectorised format choose appropriate size of figure in the document that will contain the figure.
3. Use the export button (Clipboard, ToFile(img), ToFile(vect)), select the file end execute the export into the document.
4. Often the export result will not be as you would like. The labels will be too small or too big, the lines too narrow or too wide. At this stage correct the Label size, Lines width, and Point size.
5. Repeat step 3 and to obtain the desired result.

Constructions can be exported as GeoGebra static files (and not constructions). Such files can be read in GeoGebra, eventually modified there, and exported from GeoGebra to other formats.

[^1]
### 7.5 Labelling points

Labelled points play an essential role in the analysis with OK Geometry since all lines, circles, conics, angles etc. 'passing through' the displayed labelled points are taken into account in the analysis. The labels of points should conform to some simple rules as described in Section 5. The labels of points may be set at the time of making a construction or just before OK Geometry begins with the analysis. All points that are potentially important for the analysis should be labelled.

You can access the commands for labelling points in three ways:

- Use the button for labelling (a button with the current label inside). A letter $\mathbf{P}$ in the (pressed) button will label an unlabelled point as $P$, and will delete the label if the point is labelled. A letter $\underline{\mathbf{P}}$ button will ask you for a label to be applied. (See the Sketch Editor Reference for details.)
- By right clicking anywhere on the visualisation pane. In the contextual menu click the Labels subcommand and then choose the appropriate labelling command (see below).
- When in Sketch mode - you can also select the appropriate commands in the editor menu (e.g. Action/Labels in the leftmost group of commands in the editor menu.

Note: In order to apply a labelling command, first choose the command, then apply the command to (one by one) points you plan to label, re-label, or un-label.

Note: Any change of labels may cause inconsistencies with the found relations in a previous analysis.
Here is a brief description of the commands for labelling points.

| Command | Meaning | Comments |
| :---: | :---: | :---: |
| Auto label | Let the computer choose the label of a selected point. <br> Clear the label of a labelled point. | The point you click on is labelled with the first available letter after the last applied label. <br> If the selected point is already labelled, its label is cleared. <br> To auto-label more points just click them one by one in order you want them to be labelled. <br> The label of the first point can be set with the command Label vertex. <br> Note: A quick way to manually set the next label to be used is to press Alt+S. |
| Label vertex | Choose the label for a selected point <br> Change the label of a selected point | First click the point you want to label or change the label. Then write the desired label of the point. The program will not accept an improper label. <br> To manually label more points repeat the operation for each point or skip to Auto label. |
| Label all | Label all unlabelled displayed points | All shown unlabelled points will be labelled in the same order as they were defined. Each such point will be assigned the first available label. Existing labels of points will not change. |
| Clear label | Delete the label of selected points | The command deletes the label of the point you click on. To delete the labels of more points just click on each of these points. |
| Clear all labels | Delete all labels | The command deletes the labels of all points. After this operation all points in the construction are unlabelled. |
| Anchor | Position the label | The positions of the labels can be modified only in the Sketch mode with the Anchor command or the i button (afterwards try also the mouse scroll button). Labels can also dragged around the referred point. |

## Labelling strategies

All points that are potentially important for the analysis should be labelled. Points that are clearly irrelevant should not be labelled: this can be achieved by hiding the points or clearing the labels. Below we list three simple strategies for labelling the points in geometric constructions.

Strategy 1.

Set the labels during the creation of a construction.
If points were properly labelled only small changes (with the above mentioned commands) are eventually required before the analysis with OK Geometry.

## Strategy 2.

Clear all labels in an imported construction.
Using Auto-label pick one by one in appropriate order the vertices you want to consider.

## Strategy 3.

Clear all labels in a construction.
Label all points using the Label-all command.
Apply the command Clear-label to points that are not relevant to the analysis.

### 7.6 The text of a task

A geometric construction is usually accompanied with a text that describes the construction or the task related to the construction. Such a text can be written in the dynamic construction (if imported) as a common text. It can be also written or edited at any time in the Task section of the information pane. The text may consist of the common text, but - in a way - it is possible to include in the text also symbols, mathematical formulae, and pictorial information.

## Inserting symbols in text

To insert mathematical or other symbols in the text use the symbols table available with the $\perp$ button on the title line (Figure 24).


Figure 24

## Inserting formulae and pictorial information in text

Images and formulae can be copy-pasted in the text using the command available with a right click on the text region. To obtain the desired size of the picture in text set the paste size accordingly.


Figure 25
Mathematical formulae can be imported using an appropriate editor. Apart from the pictorial format of the formula, which is imported as image, it is possible to import also the source of the formula. The source has no function in OK Geometry, except that can be retrieved in case one would like to make changes (in the appropriate editor).

## Inserting links

Links to files and web pages can be inserted with the Attach command in the context menu (right click on the text region.

### 7.7 Inspecting the construction steps

It is possible to visualise the sequence of construction steps of a construction. To do this use the command Commands/Construction steps. The same command displays the scenes of a construction.

### 7.8 Saving and retrieving constructions and projects

OK Geometry data stored data in two types of files. Current constructions are stored in files with file ending .p, while current projects are stored in files with file ending .pro.

Saving constructions. To save the current construction to a file, use the command File/Save construction in the menu. The construction is saved in OK Geometry format. Note that the saved construction contains also the information about the Task and Treatment section in the information pane but does not contain any information about icons and found or considered properties.

Retrieving constructions. A saved construction can be read with the File/Open construction command from the menu or with the Task button. By default, reading a task also clears the current project (i.e. all icons. However this feature can be blocked by checking the configuration option Configure/General options/Preserve project on importing construction.

Saving and retrieving projects. Current projects containing the found and selected properties, icons with comments and underlying dynamic constructions are saved and retrieved in files with file ending .pro . To save the current project use File/Save project command in the menu. The work that was saved
in a .pro file (i.e. the geometric construction together with the analysis, icons, and all your comments) can be retrieved with the menu command File/Open project.

### 7.9 Recovering current work

Recovering current work. OK geometry continuously saves the current work. The last saved work can be recovered with the Configure/Recover last session menu command.

## 8 Analysing constructions

In most cases there is no need to configure the analysis. Simply press the Observe button in the main menu bar. Occasionally, however, simple adjustments will be helpful.

### 8.1 Setting the points to be considered in the analysis

## Why setting the points for the analysis?

OK Geometry in a construction considers only the labelled points (see Section 5). Thus it is important that all potentially relevant points are labelled. However, an excessive number of labelled points creates several problems: OK Geometry may find so many instances of properties that it would be difficult to identify the interesting ones among them, the detection may take a long time, and the detection of certain properties may be unreliable. Thus, it is advisable to restrict the analysis to 4-12 points and, eventually, repeat the analysis on various sets of points of the same configuration. For this reason OK Geometry allows to set which points to consider in the analysis without changing the construction itself.

## How to set the points for the analysis?

The points to be considered in the analysis are set in the Treatment section of the information pane. The points to be considered in the analysis should be written in the entry Consider only points. If the entry is empty then all labelled points are considered in the analysis, otherwise only the specified points are considered. If there is an exclamation point at the beginning then the indicated points will not be considered in the analysis.

| Task | Sketch | Observe | Project | Report |
| :---: | :---: | :---: | :---: | :---: |
| $\nabla$ Task |  |  |  | 1 |
| $\Delta \quad$ Treatment |  |  |  |  |
| Consider only points: Invariant cycle 1: Invariant cycle 2: |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Level of | ervation a |  | Medium | - |
| $\triangle$ |  | sformations | meters |  |

Figure 26

Here are some examples for the entry Consider only points:
empty - All labelled points in the construction are considered in the analysis. This is the default assumption.
$A^{\prime} C 1(D G) R \quad$ - Only points $A, B^{\prime}, C_{1}, D G$, and $R$ are considered.
!ADH - All labelled points except A,D, and $H$ are considered.
Since it is very important to be aware which points are considered in the analysis, this information is always displayed as part of the title of the Treatment section.

The considered points are saved as part of the geometric construction. When you retrieve a construction (in OK Geometry format) you retrieve also the information which point should be considered in the analysis.

## Strategies for setting the considered points

Strategy 1.
The most obvious (and most common) strategy is - to do nothing and consider all labelled points. Unless OK Geometry refuses look for certain properties (since there are too many instances to check) or the list of found instances of some property is too long there is no reason to work differently.

## Strategy 2.

If there is a considerable number of labelled points in the construction, it is advisable to select an appropriate set of, say, 8-12 points, do the analysis and select the relevant properties, then repeat this for another set of points, and so on.

### 8.2 Symmetries in geometric constructions

Using this advanced feature we put additionally structure in the set of detected relations, so that they are much more reliable and also easier to manage. We can apply this feature when in a construction we expect or demand that the sought properties are invariant under certain rearrangements of vertices. It is possible to choose between two types of rearrangements:

Cyclic rearrangements. We use the notation like $A B C$ DEF to call for properties that are preserved if the vertices $A, B, C, D, E, F$ are changed cyclically to $C, A, B, F, D, E$ or to $B, C, A, E, F, D$. In other words we demand the invariance under any cyclic permutation of $A B C$ and the respective cyclic permutation of $D, E, F$.
Complete rearrangements. We use the notation like *ABC DEF to call for properties that are preserved if the vertices $A, B, C, D, E, F$ are rearranged to $C, A, B, F, D, E$ or to $B, C, A, E, F, D$ or to $B, A, C$, $E, D, F$ or to $C, B, A, F, E, D$ or to $A, C, B, E, D, F$. In other words we demand the invariance of properties under any permutation of $A B C$ and the respective permutation of $D, E, F$.

Let us explain this in with some simple examples.
Given is a triangle $A B C$. The inscribed circle (centered at $G$ ) to this triangle touches the sides of the triangle in points $D, E$, and $F$ (see Figure 27).


Figure 27
We can identify several properties in this configuration. For example, there are 6 right triangles:
AEG, BDG, CEG,
AED, CEG, BEG.
You can easily see that these triangles refer to basically the same property. We can reduce the 6 instances to just one, for example AEG. All the other instances are obtained by all permutations of the vertices $A, B, C$ and respective permutations of $D, E, F$. We symbolically describe this invariance by writing *ABC DEF in the entry Invariant cycle 1 (Figure 28). As a result, OK Geometry spots, for example, 6 right triangles. Note that 5 instances of the right triangles are written in blue colour, indicating that these instances are, basically, 'repetitions' of some other instance of property. The blue coloured instances of properties are thus obtained from other instances properties by applying the permutations in the Invariant cycle entry or entries.


[^2]Consider now a more sophisticated example: a kite (Figure 29). The kite has a line of symmetry, thus all properties are preserved if $B$ and $D$ are interchanged. Setting the entry Invariant cycle 1 to BD would therefore reduce the number of relations. Note that the geometric properties are preserved also if $A$ and $C$ are interchanged independently of $B$ and $D$. Thus setting the entry Invariant cycle 2 to $A C$ would still reduce the number of relations.

What would happen if the Invariant cycle 1 was set to ABCD and the Invariant cycle $\mathbf{2}$ left empty? The property that the triangle $A B D$ is isosceles would not be listed since the triangle BCA (obtained by a cyclic permutation of the vertices) is not isosceles. However, the fact that the line $A C$ is perpendicular to $B D$ would be listed it is clearly invariant under the cyclic permutations of vertices $A, B, C, D$.


Figure 29
Some additional notes:

- By default the Invariant cycles entries are empty, thus OK Geometry does not assume that properties should be invariant under any permutation.
- It is perfectly ok if only one invariant cycle is considered.
- The syntax of each entry is evident from the examples above. Note that each entry should consist of one or more cycles of the same length. A given point can be present in an entry at most once.
- You can write the labels to the entry either directly (using the keyboard) or by clicking the point on the construction (in the drawing pane) while the cursor is in the entry.
- Since it is very important to be aware of the invariant cycles they are always displayed in the title of the treatment section.
- The invariant cycles are considered as part of the construction. Thus they are saved together with geometric constructions. When you retrieve a construction (in OK Geometry format .p) you will also retrieve its invariant cycles.


### 8.3 Setting the level of analysis

OK Geometry looks for several properties of objects; some of these properties are of interest only for advanced users. For this reason the user can set the level of analysis performed by OK Geometry. The level of analysis can be set globally (as a default level, see General options in Section 13.2Configuration Commands), and can be adjusted locally for a particular task. You can choose among these possibilities: Simple, Medium, Advanced, and Complete.

The default level of expertise is set in the menu command Configure/General under the entry Default level of analysis.

The level of analysis for the current task is set in the Treatment section of the Information pane, as shown in Figure 30.


Figure 30
Here is a very rough description of the levels of expertise:

| Level | Rough description of the properties that are considered at given level |
| :--- | :--- |
| Simple | Basic properties of geometric objects, specific shapes (of triangles), congruence |
| Medium | Simple + same ratio of lengths and angles, simple ratios of lengths, similarity |
| Advanced | Medium + advanced ratios of lengths, points on conics, golden ratio, cross-ratio |
| Complete | Advanced + relations that may give rise to a big number of properties (e.g. pairs of <br> angles with the same sum) <br> This level is usually avoided. |

Precise information about which properties are considered at a particular level is available via the menu command Configure /Geometric relations. To configure the level User, apply the menu command Configure/Considered properties.

Choosing the level of expertise is, obviously, a matter of user's knowledge of geometry and of the content of the task. In general, choose among Simple, Medium, and Advanced level. The Complete level should be used with care and only in special cases, since the relations, specific to this level, often produce an unmanageable number of properties.

Note. OK Geometry Plus version spots considerably more properties, which are of interest only for specialists in triangle geometry.

## 9 Detecting properties

### 9.1 The list of found properties

The main feature of OK Geometry is the ability to detect geometric properties in a geometric construction. This is achieved with the Observe button in the main menu bar or with the Commands/Observe command in the main menu. The command Observe explores whether the objects (determined by the labelled points) in the current geometric construction have certain geometric properties.

The Observe command produces a list of properties and instances of properties of the studied geometric configuration. The list is rather self-explanatory, but some used conventions should be mentioned. Figure 31 shows an example of a list produced with the Observe command. This command finds out whether the considered properties hold for some (selected) labelled points of the configuration. For each property there are 4 possible outcomes:

No instance of a property was found. In this case the property is not listed. In Figure 31, for example, no right triangles were found, since the property is not listed.
Some instances of a property were found. In this case the property is listed with an expansion sign ( $a+i n$ a small square) in front of the property and the number of instances after the property. In Figure 31, for example, OK Geometry has found 4 instances of equilateral triangles. To see the found instances just click the expansion sign.
The property was not checked. In this case the property is listed without the expansion sign and the property is followed with a question mark. In the considered example (Figure 31) this is the case for the equality of ratios of areas of triangles. There are two reasons why OK Geometry would not check a property. Maybe there are so many instances of properties to check that the computation would take too much time. It is also possible that OK Geometry has found that the results of the checks would be too unreliable. There are at least two ways to overcome such a situation:

- The preferred way is to reduce the number of involved vertices (see 8.1).
- You can increase the limit for the allowed number of instances of properties to be checked (see Section 13.2). Then repeat the Observe command.

| $\Delta \quad$ Treatment |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Consider only points： Invariant cycle 1： Invariant cycle 2： <br> Level of analysis： |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  | Medium |  |
| Filter | Observed properties |  |  |  |
| 田－points（1） <br>  <br> 田 $■$ similar triangles（1） <br> 田 $\quad$ congruent segments（4） <br> 田 $■$ equal or supl．angles（1） <br> 田 congruent angles between lines（19） <br> same area of triangles（relation）（？） <br> －same ratios of areas of triangle（？） <br> congruent circumcircles（relation）（？） <br> congruent inscribed circles（relation）（？） |  |  |  |  |
| $\nabla$ Messages |  |  |  |  |

Figure 31
Notice the coloured squares in front of the names of properties．The meaning of the square is as follows：
A green square before a property indicates that the probability that instances of this property were found by chance is extremely low．

A yellow square indicates that indicates that the probability that instances of the property were found by chance is very low．

An orange square indicates that the there is a small probability that some listed instances of the property are a result of chance．The instances of such properties should be considered with due doubt．

A red square indicates that $O K$ Geometry has detected a suspicious feature in the construction that may jeopardise the validity of the results．

If you do not want to display doubtful properties，then uncheck the item Check validity of results in the main menu Configuration／General（see 13．2）．

The reliability of the results，obtained with OK Geometry，is significantly improved
－if symmetries in construction are considered（see Section 8．2）or
－if the number of considered points is reduced or
－the perturbation radius is increased（see Section Napaka！Vira sklicevanja ni bilo mogoče najti．）．

On the other hand，the observed results are less reliable
－if the construction is based on optimisation or implicit constructions or
－if the number of considered points is big．

## 9．2 Instances of properties

If a there is an expansion mark in front of a property，then the instances of the property can be displayed by clicking the expansion mark．The interpretation of the listed instances of properties is often
straightforward, especially if their graphic representations in the visualisation pane are considered. In the situation presented in Figure 32, for example, OK Geometry has found three instances of pairs of congruent triangles.


Figure 32
The way of expressing properties may appear strange, but is easy to understand, especially if the properties are visualised on the right pane. Just keep in mind two basic rules:

- Objects are represented by juxtaposition of points that define them. From the name of the property we infer the type of the involved objects (e.g. collinearity is clearly about points on lines, similar triangles is clearly about triangles). The objects are then identified by points that define the objects. For example:
- Lines are denoted by two or more points on a line (e.g. AC, AGTU). Note that the order of points does not reflect the order of the points on the line.
- Circles are denoted by 3 or more points on the circle (e.g. ABC, ABMR). Note that the order of vertices does not reflect the position of the points on the circle.
- Triangles are denoted by the three vertices (e.g. ABC, AMP). The order of vertices has no meaning.
- Angles between lines are shown as $A B \# C D$ (in this case the angle between the line $A B$ and the line $B C$; here the vertex of the angle needs not be a part of the construction). Similarly $A B C D \# E F$ represents the angle between the line containing the collinear points $A, B, C, D$ and the line $E F$. Also in this case we do not distinguish between an angle and its supplement. More precisely: ABCD\#EF denotes the un-oriented angle (between 0 an 90 degrees) between lines ABCD and EF.
- Each property is about objects of the same type. Usually the presented objects have the same property. However they are listed using separation marks of different hierarchical order. In practice you need not bother of the meaning of the separations symbols - they are self-evident, especially when we visualise the relation. The meaning of the separation marks (in order from high precedence to low precedence) is:
| parallel lines, congruent segments, congruent triangles, etc.
\# angle sign

```
~ same quantity (e.g. angle, area, perimeter), similar triangles
+ sum of quantities (lengths, angles, etc)
    : ratio of quantities, also tangency
    , enumeration of objects
    = same numerical values (e.g. same sum, same ratio, etc.)
    ; separator of enumerations
```

Here are some examples:

| Similar triangles: | In this case the triangles $A B C, A P D$, and EFG are similar, but $A B C$ <br> and APD are even congruent. Note that the order of vertices <br> means nothing, i.e. $A B C \mid A P D$ does not mean that $A B$ is congruent <br> to AP. |
| :--- | :--- |
| Sum of lengths: <br> $A P\|A C+E F=C D+C E\| B C$. | In this case the segments AP and AC are congruent, and so are the <br> segments CE and BC. Furthermore the lengths AP+EF, AC+EF, <br> $C D+C E, ~ C D+B C ~ a d d ~ t o ~ t h e ~ s a m e ~ s u m . ~$ |
| Same angles: <br> $A B\|C D \# E F\| G H[=30 \mathrm{deg}]$. | In this case OK Geometry has found an angle of 30 degrees. <br> Actually there are two pairs of parallel lines AB is parallel to $C D$, <br> and EF is parallel to GH. The angle between the directions of these <br> lines is 30 degrees. |

### 9.3 Visualising relations

To visualise a relation, just click on the displayed instance of property in the Information pane (see Figure 16). For simple properties an iconic representation with obvious interpretation will appear in the visualisation pane. For complex properties (e.g. when a property involves many objects) you will need to visualise smaller groups of involved objects or even single objects. There are two easy ways to achieve this:

- When a property is visualised, note that the property is written also in the visualisation pane, just above the drawn construction. Click with the mouse on an object in the written representation, and the object will be highlighted. If you click again on the same object then the related groups of object will be highlighted (see Figure 16).
- A more flexible way of visualising complex relation is via direction keys on the keyboard. With the keys Up and Down you select whether to represent a broader or narrower group of related objects. With keys Left and right you can move between groups of objects. Try also to combine the left and right direction keys with the Shift key (see Figure 16).


### 9.4 Query objects

After you obtain a geometric construction and press the Observe command in the main menu bar, OK Geometry gives you a list of observed properties. Some detected properties turn to be obvious or irrelevant, but some may be not. The list of all found properties is sometimes very long (e.g. several hundreds of them). If you are en experienced (advanced) user then you will certainly not go through all the properties. Instead you will focus on certain properties and not loose time with other ones. On the other hand, you are looking for properties related to a specific object. For example, you may say to yourself: "I feel that this triangle ABK is important. I wonder if it is congruent, similar or otherwise related to some other triangle." Or: "I'd like to know how to construct a point if only some of the objects in the configuration were known." You can find such information easily using the query commands.

Objects in a configuration can be queried when the observed properties are listed on the left pane. The query buttons are positioned on the task bar on the right (Figure 33, right). The query buttons from up to down are:

| Command | Meaning |
| :---: | :---: |
| Mark/unmark nongiven objects | This command is used to mark objects that are considered as non-given. Object marked as non-given are pale-green coloured. Such objects are not considered in the query, unless they are the object of query. A typical use of this function is the study of construction problems. In a complete construction we mark objects that need to be constructed from non-marked given objects. |
| Query labelled point | Click on the point to be inspected. On the left pane only properties related to the inspected point are listed. |
| Query-Segment | Click on the segment to be inspected or its endpoints. On the left pane only properties related to the inspected segment are listed. |
| Query-Line | Click on the line to be inspected or on two labelled points on it. On the left pane only properties related to the inspected line are listed. |
| Query-Circle | Click on the circle to be inspected or on three labelled points on it. On the left pane only properties related to the inspected circle are listed. |
| Query-Triangle | Click on the labelled triangle vertices. On the left pane only properties related to the inspected triangle are listed. |
| Query-Angles | Click on the labelled points (ray, vertex, ray) or on the rays of the angle to be inspected. On the left pane only properties related to the inspected angle are listed. |
| Query-ExpandAll | On the left pane all observed properties of the studied configuration are listed. |
| Query strictness | The level of the query can be ? $1, ? 2, ? 3$ or ? 4 and is set on the top bar (Figure 33 , left). At level 1 we obtain extensive information - all properties of all objects in any way related to the inspected objects are listed. On level ? 4 the obtained information is minimal - often the inspected object is related to just one (marked as given) object. |

On the top bar there is a ? button, which is used to control the highlighting of non-given objects. By default the non-given objects are highlighted, i.e. pale-green coloured.


## Figure 33

## An example

Given is a line segment $A B$ and a point $C$ on it. Consider three circles with diameters $A B, A C$ and $C B$ with respective centres with respective centres F, D, E ( Figure 34). The upper half of the obtained shape is often called arbelos. We would like to inscribe a circle in this figure, as shown in Figure 34. OK Geometry Sketch editor allows to position a circle that touches three given circles, the construction is examples directory (Example5.p). However, we are looking for a Euclidean construction. For the construction i suffices, for example, to obtain the three touch-points $\mathrm{H}, \mathrm{I}, \mathrm{J}$ (there are other options, of course).


Figure 34
Figure 35


1. First use the Observe command to obtain the properties of the configuration that includes the points $\mathrm{H}, \mathrm{I}, \mathrm{J}, \mathrm{K}$, and the inscribed circle.
2. We mark the points $\mathrm{H}, \mathrm{I}, \mathrm{J}, \mathrm{K}$, and the inscribed circle as non-given objects. since we do not yet have an idea how to construct them from given objects. The non-given objects become pale-green coloured.
3. We query a non-given point, for example the point J, with the command query point. We obtain a restricted list of properties that involve the point J, but not H, I, K and the inscribed circle.
Some of the properties are related to the fact that J lays on the circle centered at E . However, some properties give a clue (not proof!) to the construction task. For example, the property 'ratio of distances' $\mathrm{AC}: \mathrm{AB}=\mathrm{CJ}: \mathrm{BJ}$ allows the construction of $J$ using the Apollonius circle in triangle BCJ.

Perhaps the nicest property is related to angles: The angle $\angle$ AJC measures $45^{\circ}$. This observation leads to hypothesis that $J$ is the intersection of two circles (Figure 36), one of them has $L$ as centre and passes through $A$.
4. In a similar fashion we hypothetise how to construct point I, J, K. Not that once we have an idea, how to obtain J, we can consider J as given etc. Needles to say that the observed properties are mere hypotheses and should be proved.

## Note.

- Try different query levels. In construction problems the best level are ?4 and ?3.
- In construction tasks pay attention to 'multiple intersections' properties. Sometimes it is possible to obtain from these properties unusual and unexpected ideas.


### 9.5 Adding new displayed objects to the construction

A visualised relation sometimes contains objects that we would like to be part of the construction. For example, OK Geometry detects that four points lay on circle and displays this circle. If we would like to include the displayed circle as a part of the current construction, we can achieve this in two ways:

- If we press the red + button above the visualisation pane (see Figure 16), the added object will be displayed as emphasized objects.
- If you want to set the style of the added objects (e.g. thin blue dotted lines), apply the $ঞ b$ button, select the type of line, and then apply the red + button (Figure 37).,


Figure 37
In any case, adding to construction objects that depict a property brings us to the Sketch editor. In the Sketch editor it is sometimes necessary to make some adjustments to the added and emphasized objects:

- Use the Anchor button (possibly together with the mouse wheel) to position/select an angle between lines and to move around the point labels.
- Use the Colour/Style button $\bullet$ to change the appearance of objects.
- You may also want to use Hide button or Delete button $\mathbf{x}$.

Figure 38 shows an example of adding object from a displayed relation. The detected relation (left) was added with the + button to the construction (center). The representation was then modified (note the position of angles and labels) with simple editing commands Anchor, Delete, Change colour/style..


Figure 38

## 10 Editing properties

### 10.1 Declaring the construction as an icon

If a construction or a represented relation (instance of a property) is for some reason relevant, e.g. to be eventually used in a proof or included in a report, we can iconise it and submit to a further elaboration. To do this:

1. Add (and eventually modify) the relation related objects to the construction (see Section 9.5).
2. Declare the current construction as an icon. This is done by pressing the button $\rightarrow$ above the visualisation panel. Declare the current construction as a New icon or overwrite an existing icon.
3. At this stage (or later) it is also possible to comment the icon. See Section 4.2 (Figure 15, Figure 18) for an illustration of this process.

### 10.2 Retrieving the construction from icons

All declared icons consist of a construction which can be retrieved with the $\square \rightarrow$ button above the visualisation pane. The Task icon can be retrieved either with the $\boldsymbol{\square} \rightarrow$ button.

## 11 Menaging the project

Once the relevant properties are iconised, you can proceed to the next step: explore the connections between the properties. The nature of these connections depends on the intention with which they were selected. A possible intention is simply to collect interesting facts related to a geometric configuration. But facts related to a geometric configuration are not unrelated: some facts may be deduced from others and form up a logical structure. Proving a fact means finding deductive arguments
from premises via known facts to the fact to be proven. Establishing deductive connections between relations of a geometric construction is a step from observing to knowing.

To organise the icons press the Project button in the main menu bar. On the left side you will see the marked properties, on the right you will see, one on the top of other, the iconic representations of the marked relations. These representations can be dragged around. After some elaboration you may get something like shown in Figure 39.


Figure 39
It is up to you to decide about the purpose of selecting relations to be shown and of the content of the elaboration. Here are some ideas for a possible content:

- relations and comments organised in a proof of a fact related to the considered construction;
- a scaffold for a proof to be used by students as an aid;
- a list of properties of the construction you found interesting, a memo;
- a set of tasks for the students (do not abuse!).


### 11.1 How icons are structured

Icons are iconic representations of relations. Each icon consists of :

- The underlying construction. This is the construction on which the relation was detected.

Different icons may be based on different constructions. The underlying construction of the icon can be retrieved using the context menu (right click on the icon and select the command Set as construction). In this way it is possible to work simultaneously with several different constructions.

- The enumeration. If the icons are organised in a linear structure, then they are enumerated with numbers $1,2,3, \ldots$ In this case all icons belong to a same level. However icons can be demoted to a sub level or promoted to a higher level. A demoted icon becomes a 'sub-icon' of
the previous icon of the higher level. For example, the icon no. 2 may have sub-icons 2.1, 2.2, 2.3 , etc. The Icon 2.1 perhaps contains sub-icons 2.1.1, 2.1.2, etc. In this way the icons cam be organised in a tree-like structure.
- Other information. Information of other kinds can be attached to an icon by editing it. In particular, the name (property) of the icon, a comment can be added to the displayed icon.


### 11.2 Organising and commenting relations

Working with the relations and their iconic representations is rather straightforward. Most operations are also accessible via the context menu (right click):

| Position the icons | Drag the icons on the visualisation pane. You may also arrange all icons in a cascade or array (see the command below). |
| :---: | :---: |
| Change icon number | Write the number that precedes the desired icon number. For example, if you want that the icon number follows the number of the icon 2.3 , write this number in the form that appears. <br> Note that the icon number 1 (Task) cannot be renumbered. <br> You can change the numeric order of the icons by dragging the icon names on the left pane or using the + and - key. |
| Edit icon name <br> Edit comment <br> Visual elements and formulae in comments | Double click the related icon (on the visualisation pane) or double click on the relation (left pane). In the form that appears you can modify the comment or the name of the icon. <br> Comments may contain information of various types. In case the comment is a deductive argumentation, it is possible to structure it by specifying the status of paragraphs (with a right click on the paragraph). A paragraph may be given the status of <br> - definition/assumption/declaration <br> - claim/hypothesis <br> - argument/reason of truth <br> Furthermore, a paragraph can be formatted as a comment or a formula. <br> See Figure 44 for an example of a structured comment. <br> Simple mathematical expressions can be written with symbols available with $\perp$ button. Complex formulae and images in general can written in other software and copy/pasted to OK Geometry - use Image\|Paste image command in context menu (obtain with a right click). |
| Edit construction | This command sets the icon's underlying construction as the current construction in the Sketch editor. The edited construction can be saved as a new icon or as an existing icon. |


| Modify the appearance of the icons | Use the sliders at the bottom of the left pane to modify the size of all icons, labels or angle notation, visual emphasis of the relation, and the bleaching effect. |
| :---: | :---: |
| Delete an icon <br> Deleting a relation | Right click the icon or the relation and select the Delete command. Deleting a relation deletes also the related icon and vice versa. |
| Arrange the icons | Icons can be spread on the pane in various ways: <br> - in the form of an Array, so that the content of icons is visible, <br> - in the form of a Cascade, so that they are all well visible; <br> - in a Free form, so that they can be moved around on the visualisation pane, <br> - in the form of a Flowchart (this form is still under development). <br> The default form is the Array. |
| Promote or demote an icon | Icons belong to up to three detail-level. <br> Icons at detail-level 1 are numbered as: 1, 2, $3 \ldots$ Those at detail-level 2 are numbered as 1.1 or 2.4. Enumeration like 2.1.4 belong to icons of detail-level 3. <br> Demoting an icon increases its level, promoting decreases the level. <br> To promote / demote an icon highlight the icon (or the respective relation) and press F2 for promotion and F3 for demotion. The commands can be accessed by a right click on the icon or relation. |
| Set the displayed detail-level | If the displayed detail-level is set to 2 , only icons of level 1 and 2 will be displayed. The displayed detail-level is set on the navigation bar just below the main menu bar (see Figure 20). <br> Note. <br> - The displayed detail-level is by default set to 3 . Changing it makes sense only when presenting the work. <br> - When the displayed detail-level is 1 or 2 the sequence of icons cannot be reordered. |
| Set (icon) as construction | In complex situations we work with more than one construction. In case of a proof, for example, we need some lemma based on a different construction. In such cases we save different construction as relations. Using the command Set as construction the underlying construction of the icon becomes the current construction in OK Geometry. <br> The command is accessible in the context menu (right click the appropriate icon). |

### 11.3 Emphasised, bleached and transparent objects. Auxiliary objects

Objects in icons can possess some attributes for simple modifications during visualisation of the icons.
Emphasised objects are displayed with variable width, which can be controlled with sliders in the left pane (see Figure 19). Objects can be declared as emphasized in Sketch editor (see also Section 10.1). Emphasized object can be dynamically set as more or less prominent during the visualisation of icons, as shown in the case of circle and triangle on Figure 40.


Figure 40
Bleached objects are displayed with variable level of fading. The level of fading of the objects can be controlled with sliders in the left pane (see Figure 19). Objects can be declared as bleached in Sketch editor (see also Section 10.1). Bleached object can be dynamically set as more or less faded during the visualisation of icons, as shown in some angles and triangles on Figure 41.


Figure 41
Transparent objects are transparent (invisible) only in icons, otherwise they are coloured in yellow. Object can be transparent in some scenes and visible in other scenes of the same icon. Note that transparent objects are considered in observation while hidden objects are not. Objects can be declared as transparent in Sketch editor (see also Section 10.1).

In OK Geometry two types of auxiliary objects are available. The first one is the text object (simple text in the drawing), which can be used for clarifying a construction or for some other purpose. The second one is the marker object, which consists of a symbol ( 1 , * , ?, etc.) and a text that is visible only when the mouse pointer is positioned on the symbol (Figure 42). Markers are used for various sorts of comments to be selectively visible.


Figure 42

### 11.4 Controlling the visibility of objects

Objects possess various levels of visibility. An object can be visible or not visible, emphasised, bleached and transparent. Usually the levels of visibility are set in the Sketch mode (see Sketch Editor Reference). However, the visibility itself can be controlled also outside the Sketch mode.

In the Project view the emphasis, highlight, and bleach are controlled (some only in the view) with the sliders in the letft pane, as explained in Section 11.3. Elsewhere the buttons in the upper right can be used (Figure 43).

If button $\mathbf{H}$ is active, hidden objects are displayed as if they were not hidden.
If button $\mathbf{B}$ is active, bleached objects are displayed as if they were not bleached.
If button $\mathbf{T}$ is active, texts in the drawing are visible. However, labels of objects are still visible.
If button $\mathbf{M}$ is active, markers visible otherwise they are withdrawn.
If button $\mathbf{N}$ is active, transparent objects are visible otherwise they are not.
if button $\mathbf{X}$ is active, then auxiliary lines (related to angles) are visible.
If button ? is active, non-given objects are highlighted they are not.


Figure 43

### 11.5 Scenes

An icon may contain several relevant features. In order to emphasise various features on the same icon it is possible to set various scenes (of an icon). For each scene it is possible to specify the objects to be emphasised, bleached and/or make transparent. Scenes can be set in Sketch editor (see the Sketch Editor Reference). Scenes may be displayed in the presentation of the work (see Section 11.6).

### 11.6 Presenting the work

The commented relations can be presented one by one in the same order as in the list in the left pane. First set the level of details to be presented (in case the icons are organised in detail-levels). Choose level 1 to display only the basic points of the work, and level 3 to display all details. In order to start the presentation press the $\square$ button below the main menu. A big icon appears (Figure 44). To move between the icons use the < and > buttons on the title of the Task section. Use the >> and << buttons to skip the details. Alternatively, use the PgUp, PgDn, Home, End keys on the keyboard. During the presentation the Task and Properties section in the left pane can be shrunk or expanded. To stop the presentation, click again the
 button.

In the presentation of an icon you can visualise various scenes of the icon, if they were set. To select a scene just click on one of the scene numbers displayed above the visualisation pane.


Figure 44

## 12 Making a report

A report combines some or all icons of a project together with their comments into a document, which can be further edited with appropriate application. Note that before producing report a Ghostscript interpreter should be installed ${ }^{3}$.

OK Geometry produces documents in (essentially) three formats:
HTML format (file endindgs .htm, .html, .mhtm, .mhtml) is best viewed on computer display.
DOC format (file endings .doc, .docx) is a standard format for word processors, like Microsoft Word.
TEX format (file endings .tex) is a format used in typesetting applications.
HTML and DOC formats allow several report styles. Icons and comments can be arranged in various ways in columns or otherwise (Figure 46).

Here are the steps for generating a report:

1. Activate the Report command (in Project view).
2. Select the name of the file to contain the report. The format of the file depends on the file ending of your file.
3. In the form that appears (select the icons to be included in the report.
4. Click Advanced features and select the features of the report.
5. Click OK and generate the report.
6. Some features in the report will not be as desired (Figure 45) point labels may be too small, etc.)
7. Repeat the whole process, but set Advanced options according to resulting features in step 6.
8. Finish the report in appropriate application.

Afterwards it is possible to specify some modalities of the report to be generated (Figure 45). The most important is the style of the report. Figure 46 illustrates some of the available style.

[^3]

Figure 45



Figure 46

## 13 Menu commands

Here is a brief overview of the available menu commands:

### 13.1 File Commands

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| New project | Begin a new project | Begin a new project. Deletes all icons as well as the <br> current construction and starts the Sketch editor. |
| Open $\mid$ Open <br> construction | Open (import) an OK <br> Geometry <br> construction file <br> Import a construction <br> from Cabri, Cabri <br> Express, GeoGebra, <br> Cinderella, C.a.R. | Read a construction that was saved in OK Geometry or <br> in some other format. The read construction overrides <br> the current construction. If not set otherwise in <br> configuration also clears the current project. |


|  | (Compass and Ruler), <br> Sketchometry <br> Sketchometry, JGEX <br> or OK Geometry file |  |
| :--- | :--- | :--- |
| Open\|Open project | Open a project | Reads a project that was previously saved in a .pro <br> file. |
| Save\|Save <br> construction | Save the current <br> construction to a file | The current construction will be saved in OK <br> Geometry .p file format. |
| Save\|Save project | Save the current <br> project | The current project will be saved in the .pro file <br> format. |
| Export\|Export <br> image | Export image to <br> clipboard, raster file or <br> vectorised vile | Rest |

### 13.2 Configuration Commands

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| Working mode | Choose between the <br> Easy mode, the Basic <br> mode, and the Plus <br> mode of work using <br> OK Geometry | The Easy mode is intended for occasional users. In this <br> mode it is possible only to analyse imported <br> geometric constructions or produce reports (e.g. <br> proofs) from previously schemas. See Help for Simple <br> mode for details. <br> The Basic mode is intended for users who want to <br> have full control over OK Geometry when studying <br> geometric situations. The Basic mode encompasses a |


|  |  | powerful geometry editor (see Sketch Editor <br> Reference); all other functions are described in this <br> document. <br> The Plus mode is intended for specialist in the area. In <br> this mode some additional functions are included, and <br> a wider range of properties are studied (e.g. all <br> Kimberling points). |
| :--- | :--- | :--- |
| General options | Set the general <br> configuration data | Here it is possible to set several working options <br> (language, size of fonts, etc.). <br> Right click on the entries to obtain help for specific <br> entries. |
| Considered |  | Information about the <br> properties that are <br> checked for various <br> levels of analysis. |
| Select the level you are interested in to display the <br> checklist of the properties. Right-click the property to <br> obtain more information about the property. <br> In the form you can write additional values and <br> relations to be checked. |  |  |
| Load configuration | Load a previously <br> saved configuration of <br> OK Geometry | A configuration of OK Geometry can be stored and <br> retrieved. |
| Save configuration | Save the current <br> configuration of OK <br> Geometry | A configuration of OK Geometry can be stored and <br> retrieved. |

### 13.3 Workflow Commands

This menu contains mainly commands from the main menu bar and some special commands available only in the Plus mode.

| Command | Meaning | Comments |
| :--- | :--- | :--- |
| Conform icons | Transfer selected <br> properties of an icon <br> to selected icons. | This command also adjusts the size of the represented <br> word and/or the represented objects of the selected <br> icons so that they conform with those of the given <br> icon. |
| Slightly shake the <br> construction | Slight modification of <br> the current <br> construction (F6) | The free points in the current dynamic construction <br> are slightly moved. The movements are minor so that <br> the construction remains essentially the same. |
| Construction steps | Display construction |  |


|  | steps and scenes of <br> the current <br> construction | $\square$ |
| :--- | :--- | :--- |

### 13.4 Help Commands

The meaning of menu entries is self-evident.


[^0]:    ${ }^{1}$ A postscript interpreter in needed also for some other operations (e.g. to copy displayed constructions or to include them in reports). Ghostscript GPL Release, a free postscript interpreter, is available at http://www.ghostscript.com/download/gsdnld.html.

[^1]:    ${ }^{2}$ A free Ghostscript interpreter release is available on http://pages.cs.wisc.edu/~ghost/doc/GPL/gpl902.htm
    27

[^2]:    Figure 28

[^3]:    ${ }^{3}$ A free Ghostscript interpreter release is available on http://pages.cs.wisc.edu/~ghost/doc/GPL/gpl902.htm .

