

OK Geometry Basic - Part I

Reference for OK Geometry Basic (v.19)
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If you are new to OK Geometry, it is best to start with the introductory examples, especially Example 1 Example 2, in Section 2. We recommend continuing with Section 1 that describes the concept of automated observation of dynamic constructions and the terminology used in OK Geometry. The heart of the reference is the Section 3 that contains the list of processes in the workflow. You can focus on the processes you are interested in (e.g., how to import and observe constructions) and refer to the corresponding details.

1 What is OK Geometry?

OK Geometry is a tool for analysing geometric constructions. Given a dynamic geometric construction, OK Geometry detects patterns as well as certain properties of the construction. With OK Geometry you can also organise the information found.

OK Geometry was initially intended for students that often find it difficult to cope with geometry exercises such as construction tasks or tasks that require building up a proof. Solving such tasks requires an appropriate organisation of the properties of the configuration. A necessary step in solving many geometry tasks is thus to be aware of the properties of a configuration. OK Geometry can help students to overcome this step.

For teachers, this program can be useful for creating exercises and for presenting geometric properties and formal proofs in electronic or written format.

For recreational mathematicians interested in classical geometry, this program can be a useful tool for analysing geometric configurations, in particular those related to triangle geometry. OK Geometry contains a huge database of characteristic points, lines, circles, and conics that are used in analysing objects with regard to a given triangle.

You can think of OK Geometry as a pair of geometric spectacles. Using sophisticated numerical and non-numerical methods, OK Geometry detects potential properties of dynamic geometric constructions (e.g. that certain points are collinear or that certain triangles have the same area). **Students should be aware that one thing is to see (observe) a property, another thing is to know that a property is true, and yet another thing to have an explanation why a property is necessarily true.** OK Geometry helps the student in the first step. OK Geometry does not prove that something is true, it is even possible (though very unlikely) that it makes an 'observation error'. OK Geometry helps to observe a configuration, and this is an important step towards knowing why something in the configuration is true. OK Geometry also helps to relate a given object in a configuration to other objects (Is there anything in this configuration that is related to a given angle?). With OK Geometry you can also organise the properties found.

OK Geometry works in three modes. The **Easy mode** is intended for occasional users - it allows the analysis of geometric constructions created by dynamic geometry software (such as GeoGebra, Cabri, Cabri Express, Sketchometry, Cinderella, C.a.R. (Compass and Ruler), Sketchometry or JGEX). The Easy mode is also intended for solving exercises in deductive reasoning in planar geometry: such exercises can be prepared with OK Geometry (but not in Easy mode).

To access the **Basic mode**, click *Configure/Working mode/Basic* on the menu (Figure 1). This document considers only the Basic mode of OK Geometry, which is also part of the Plus mode.

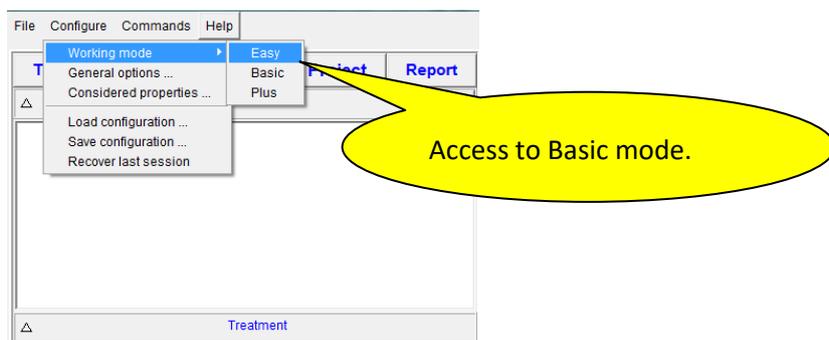


Figure 1

Here is a concise list of tasks that OK Geometry can do in the Basic mode:

- import constructions from different dynamic geometry software (DGS), currently for Cabri, Cabri II, Cabri Express, GeoGebra, Cinderella, CaR (Compass and Ruler), Sketchometry, JGEX (Java Geometry Expert);
- make new constructions or edit the imported ones;
- modify constructions in order to achieve certain goals (e.g. automatically move a point in order to obtain a maximal area of a shape in a construction);
- spot patterns and properties in geometric constructions;
- investigate properties related to a specific object in a construction;
- select relevant properties for further elaboration;
- organise the selected information (e.g. to build up a proof);
- present properties of a configuration (or a proof);
- make a report that can be read with common word processors or browsers.

The **Plus mode** of OK Geometry offers much more. First, observed objects can be related to a huge database of characteristic objects of triangles. Second, the observed objects can be related to objects that are not part of the geometric configuration under consideration. Third, you can observe algebraic relationships between geometric quantities. Fourth, you can create and observe families of constructions.

This document contains the descriptions of the basic concepts and operations of OK Geometry. A separate document describes the OK Geometry Sketch Editor for creating constructions and geometric configurations. The advanced observations and operations of the Plus mode also described in a separate document.

1.1 The model behind OK Geometry

Planar geometry is about configurations of geometric objects such as points, lines, circles etc. The objects and configurations can be considered from a naive perspective, but in geometry we consider them as abstract entities, and the properties of geometric configurations are derived from an assumed set of axioms from which a system of definitions and theorems is built. Geometric reasoning requires that geometric objects be represented in some way, whether one considers them as naive or as formal. The representations of geometric objects are usually based on a model, so that not only the objects but also the properties among them can be represented in some way. A model thus helps to visualise and understand properties and can be used to informally verify the validity of claims and hypothesise new

properties. Unlike the "paper-and-pencil" way of representing geometric objects, which is based on the static model, and unlike dynamic geometry software, which is based on the dynamic model, OK Geometry uses the stochastic dynamic model for representing geometric objects.

Before we briefly explain the three models we need to clarify an aspect of geometric constructions. Consider, for example, an equilateral triangle (Figure 2). To construct a general equilateral triangle we start from two of its vertices, say A and B, which can be positioned arbitrarily on the plane. The third vertex of the triangle, say C, has to be positioned so that its distance from A and from B is the same as the distance between A and B. We say that A and B are *non-constructed* or *free* objects, while C is a *constructed* object. The line segments AB, BC, and CA are, obviously, also constructed objects since they are determined by object set previously.

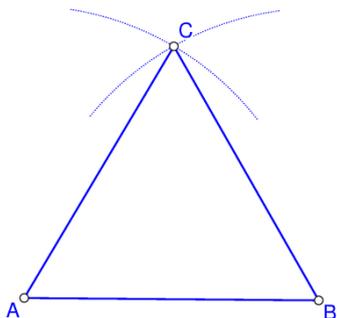


Figure 2

In the **static model** all represented objects, whether free or constructed, are fixed. For example, the plane is represented as a sheet of paper, points by drawn dots on it, etc. The drawn objects are static.

In the **dynamic model** the plane is represented, for example, by a computer display and the objects by dots or sets of dots on it. The free (non-constructed) points can be dragged. The constructed objects cannot be dragged but they change in accordance with the construction and the position of the free points.

The **stochastic dynamic model** is similar to the dynamic model. However, the free (non-constructed) objects are represented by several copies of randomly positioned objects. The constructed objects are represented by several objects constructed from copies of other objects in accordance with the construction. Only one copy of each object is visible, and assumes a reasonably random position (as well as its copies).

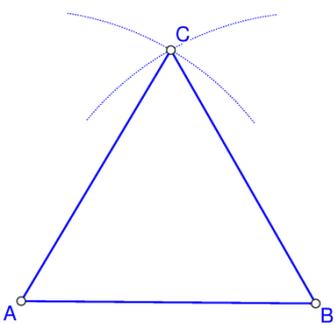
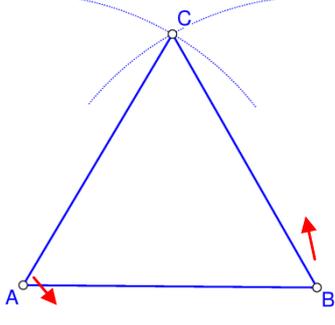
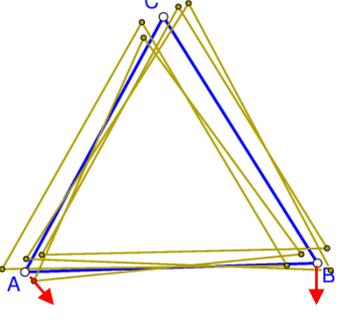
		
<p>The static model. All represented objects are fixed.</p>	<p>The dynamic model. The free points A and B can be dragged. All other objects change accordingly.</p>	<p>The stochastic dynamic model. The free points A and B can be dragged. There are several randomly positioned copies of each object – only one of them is visible.</p>

Figure 3

The three models are illustrated in Figure 3 for the case of an equilateral triangle with given points A and B. The difference between the models becomes clear when a property is checked. In the static model an observed property (based on a measurement) can easily be a matter of chance. The same is true for the dynamic model; however, by dragging, other instances of construction can be created and checked for the property. In the stochastic dynamic model a property is checked simultaneously on all random copies of the object, thus there is no need for dragging to check a property.

1.2 How OK Geometry observes properties?

From a dynamic construction, OK Geometry creates multiple instances of the construction by randomly dragging all free points. The properties that are invariant (in terms of measurement) to such perturbations (i.e., are detected by measurement in all created instances of the construction) are considered "observed". OK Geometry detects properties and provides a reasonable warrant for their truth, but does not prove them.

Here is a simple example. In GeoGebra, we use the mouse to position: point A at (1,1), point B at (3,1), and point C at (5,1). The fourth point D is constructed as the midpoint of A and B (Figure 4, left)¹. When this construction is imported into OK Geometry, OK Geometry creates several instances of the construction by dragging the free points A,B,C (Figure 4, right, shows only one of the instances). In GeoGebra the points A,B,C are collinear and B is the midpoint of A and C. But OK Geometry does not observe these properties since they do not follow from the construction. However, the observation in OK Geometry shows that D is the midpoint of A and B, since this is true by construction in all perturbed instances of the construction.

¹ OKExamples\Basic_01.p

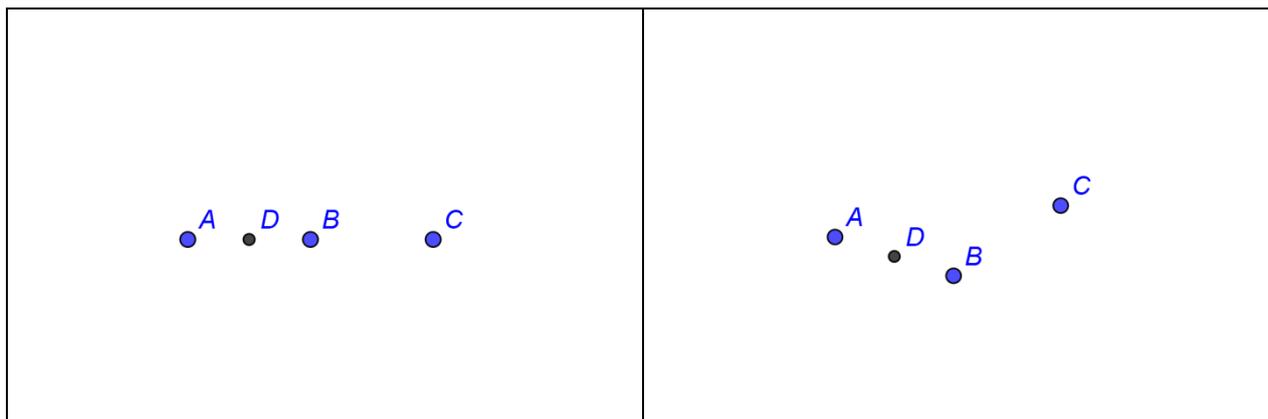


Figure 4

In the analysis of constructions OK Geometry considers

- all **displayed labelled points** (unlabelled points are ignored);
- all displayed lines, circles, and similar objects;
- all lines (displayed or not) passing through any two **displayed labelled points**;
- all circles (displayed or not) passing through any three **displayed labelled points**;
- all conics (displayed or not) passing through any five **displayed labelled points**;
- specific objects with vertices on displayed labelled points (e.g. squares, displayed or not)

The displayed points can be labelled or not, however, **only labelled points are considered in the analysis**. Remember:

- **OK Geometry considers only acute and right angles between lines.** OK Geometry usually observes something about *certain angles or their supplementary angles*. The user has to find out if an angle and or its supplementary angle must be considered. In Figure 5 (right)², for example, with reference to $\angle C$ and $\angle D$ - the ratio 1 : 2 is observed between the two 'angles or their supplementary angles'. Obviously, in this case the supplement of the shown $\angle D$ measures twice as the shown $\angle C$.
- **OK Geometry does not list trivial congruence of angles (angles related to pairs of parallel lines, angles formed by two intersecting lines)**, because each pair of parallel lines would produce an immense number of congruent angles. Therefore, when considering the observed properties one should check if parallel lines exist and be aware of the congruence of angles defined by parallel lines and transversal lines. In the trapezium (Figure 5, left)³, for example, OK Geometry does not list the fact that the angles $\angle DEA$ and $\angle BEC$ or $\angle DCA$ and $\angle CAB$ are congruent.

² OKExamples\Basic_03.p

³ OKExamples\Basic_02.p

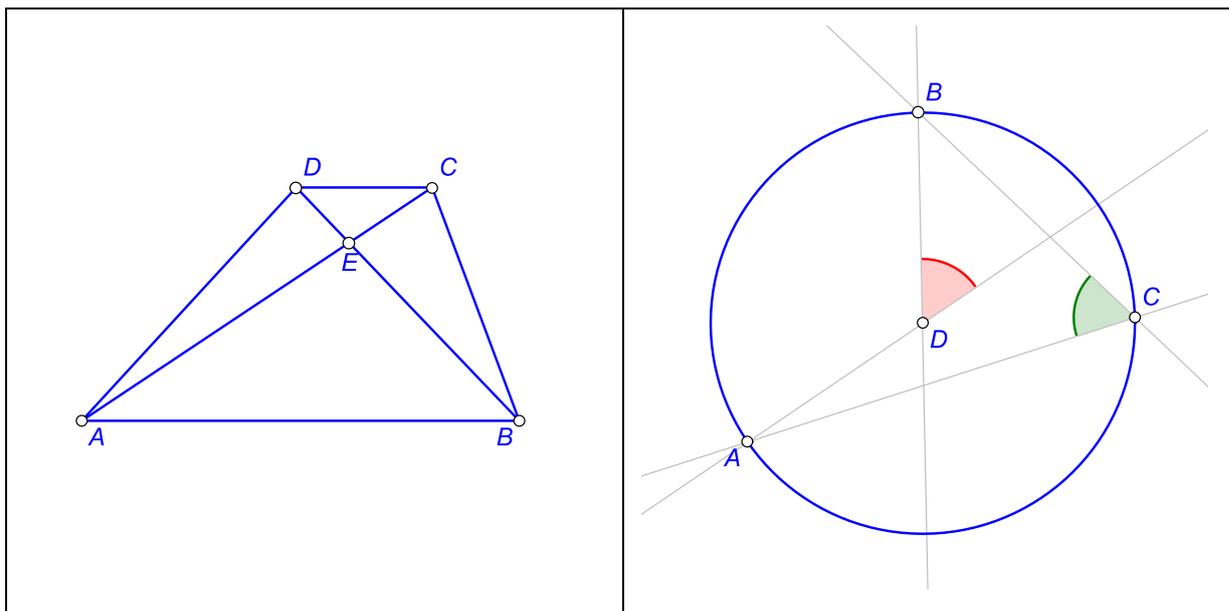


Figure 5

- **Point labels** consist of one or more letters, possibly followed by one or more digits and apostrophes. Here are some examples of allowed labels: A, B, b, P, c', D'', E13, t2', Ab. Labels like E13 or Abc are sometimes displayed as E₁₃ and (Abc). For example, the vertices of the triangle $\Delta E4(Ad)P$ are: E4, Ad, P. Labels are case sensitive. Lower case x,y,z cannot be used as labels. Different points cannot share the same label.
- **A reasonable number of labelled points to be considered in a construction is 4-12.** The number of found relations generally rapidly increases with the number of points to be considered in a construction. An excessive number of points to be considered can lead to an unmanageable collection of properties or to an unclear visual representation of the properties. During the analysis it is possible to exclude some labelled points from consideration (see section 4.2.1 on excluding points from the analysis and 4.1.5 on changing labels).
- Only properties whose number of instances does not exceeds a threshold (set in *Configure|General options|Observe*) are observed. See Section 4.2.4 for details.

The decision which points to label in a geometric construction is very important. In general, try to label all points that you find potentially relevant to your investigation. In many cases the decision which points to label is rather straightforward, but sometimes a good intuition helps.

1.3 Exact and perturbed view. The blue dot

The main task of OK Geometry is the observation dynamic constructions. When observing, OK Geometry always uses the stochastic dynamic representation of objects, so that it considers several copies of perturbed points around each given free point - although only one copy of the (perturbed) construction is displayed (Figure 6). Sometimes one prefers to visualise the construction with free points at the exact input positions (Figure 6). For this reason OK Geometry can visualise both, the exact and the perturbed view.

The blue dot on the left upper corner of the displayed construction (Figure 15) serves as an indicator as well as a switch between the exact and the perturbed visualisation. When in exact mode, the blue dot appears as gray disc with a dot in its centre  , while in perturbed mode the blue dot contains some random red dots  . The size of the blue dot (perturbation radius) shows the size of the perturbation area around free points. A **click on the blue dot** changes the modes from one to the other.

Internally, all observations are made in the perturbed mode. On the other hand, all exports of constructions are made in the exact mode, independent of the status of the blue dot.

Note that certain constructions cannot be displayed in the exact mode.

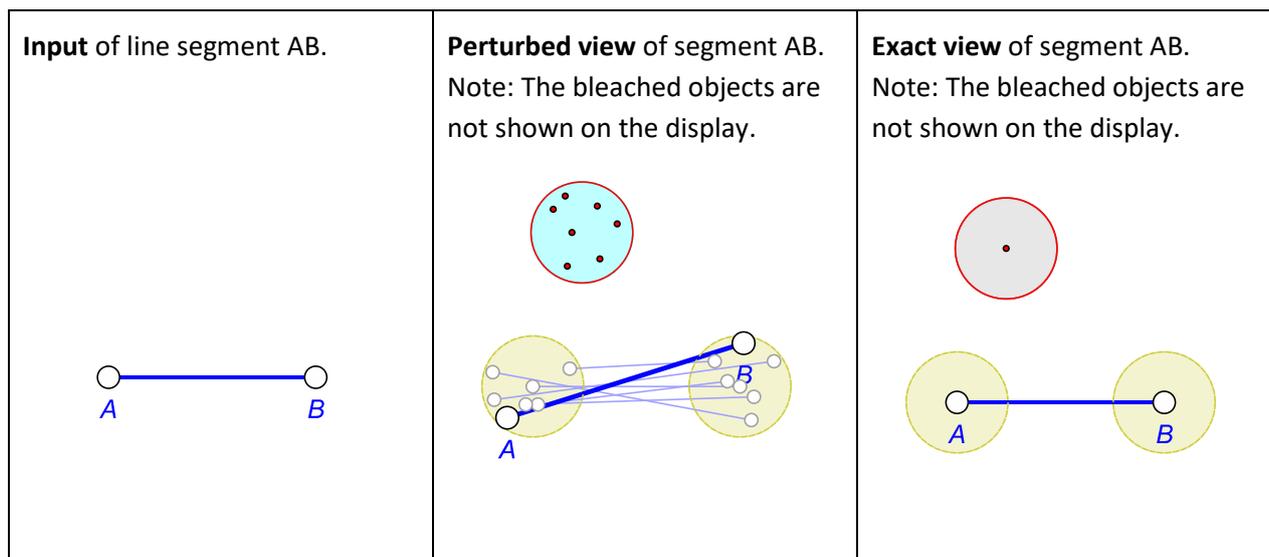


Figure 6

The power of OK Geometry is, by no means, in the perturbed representation of objects. Sometimes viewing objects in perturbed mode is not desired since points and the whole construction change slightly from time to time. Also, in perturbed mode we cannot draw exact horizontal or vertical lines. On the other hand, in the exact mode, we can draw perfect horizontal and vertical lines (OK Geometry gives a help to do this). There is also a command in Sketch Editor to visually align points.

You can switch between the exact and perturbed view with a **click on the blue dot**. Alternatively, you can press the **F7 key** or use the command *Commands/Perturbate points* in the main menu.

A **right click on the blue dot** allows you to configure some frequently used parameters related to the display of the configuration (e.g. the way of filling areas, font of point labels). In particular, you can also set the level of the displayed help for the construction commands.

1.4 Organisation of data

1.4.1 Geometric constructions and geometric configurations

In this document the terms **(geometric) object** or **entity** to denote any basic geometric object such as point, line, circle, arc, (line) segment, polyline (i.e. a concatenated sequence of line segments), conic. In

a broader sense, the terms also refer to real numbers (related to geometric quantities) and real valued variables (parameters).

By a **(geometric, dynamic) construction** we mean a set of free basic objects (points or real numbers) and a sequence of objects obtained from free or previously constructed objects using ‘allowed’ construction methods. The ‘allowed’ construction methods should be reducible to basic Euclidean construction steps.

By a **(geometric) configuration** we understand a set of geometric objects positioned in Euclidean plane, usually satisfying certain relations.

Visual representations of a configuration are called **scenes**. Each configuration has a default visualisation (called the **ground scene**), but other scenes can be added to it. The scenes of a configuration may differ from the default one only in the way objects of the configuration are more or less emphasised and more or less visible.

1.4.2 Construction view and project view

We use the term **icon** for a pictorial representation of an entity consisting of a geometric configuration and a textual description.

Several icons are organised in a **project**. For example, a project can be about interesting properties of a geometric configuration, the steps of a proof of a theorem, a related set of constructions. The basic (starting) icon of the project is named **Task**.

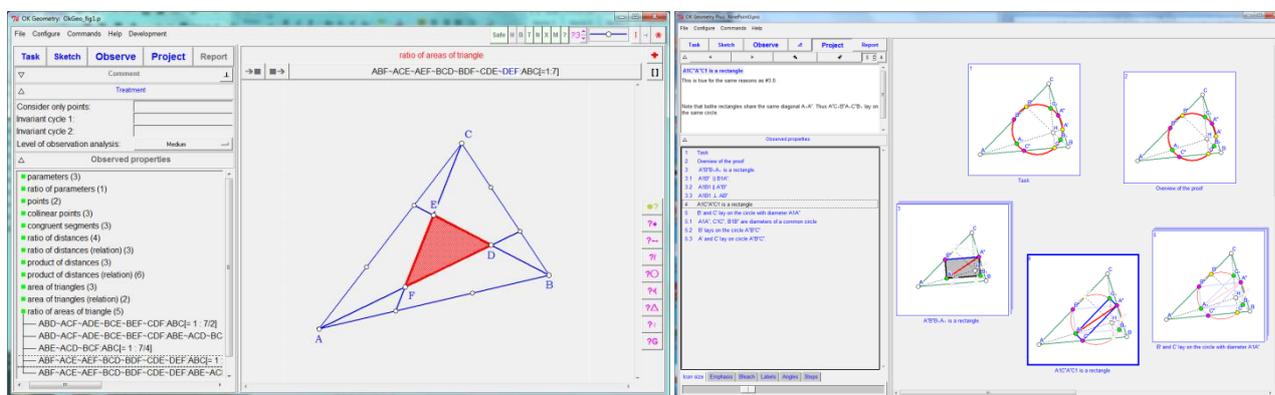


Figure 7

1.4.3 Geometric properties and geometric relations

The two central concepts we shall often use are: geometric properties and instances of properties. We shall use the term **(geometric) property** for general properties, e.g. collinearity of points, perpendicularity of lines. By **instance (of a property)** or **relation** we mean a property applied to specific objects in a configuration or construction, e.g. the fact that the points A, B, C in a configuration are collinear is a relation, thus an instance of the property collinearity.

1.4.4 Structure of project

Figure 7 shows the two types of views in OK Geometry. The left screenshot in Figure 7 is the **construction view**. On the right part of the construction view contains the visualisation of a geometric configuration, while the left part contains data about properties of the configuration (when a property is observed) or editing information (when a construction is edited).

A project is a collection of icons. Each icon contains a construction with corresponding comments and is represented with a small image. The right screenshot in Figure 7 is the **project view** of a project. The project view consists of icons (right part of the screenshot) and some icons related data (left part of the screenshot).

OK Geometry is often used in a very simple fashion (Figure 8), just to study a construction in two steps:

1. A dynamic construction is made in Sketch Editor, which is a module of OK Geometry, or imported from dynamic geometry software.
2. The geometric properties of the construction are analysed and studied.



Figure 8

You can store in OK Geometry single constructions in files with .p extension.

OK Geometry allows also a more complex schema of work (Figure 9). The work consists of a project that contains several icons, i.e. commented constructions with. A project can be about a theorem and its proof, about properties of a geometric shape, about a family of geometric objects, about a group of related theorems, etc.

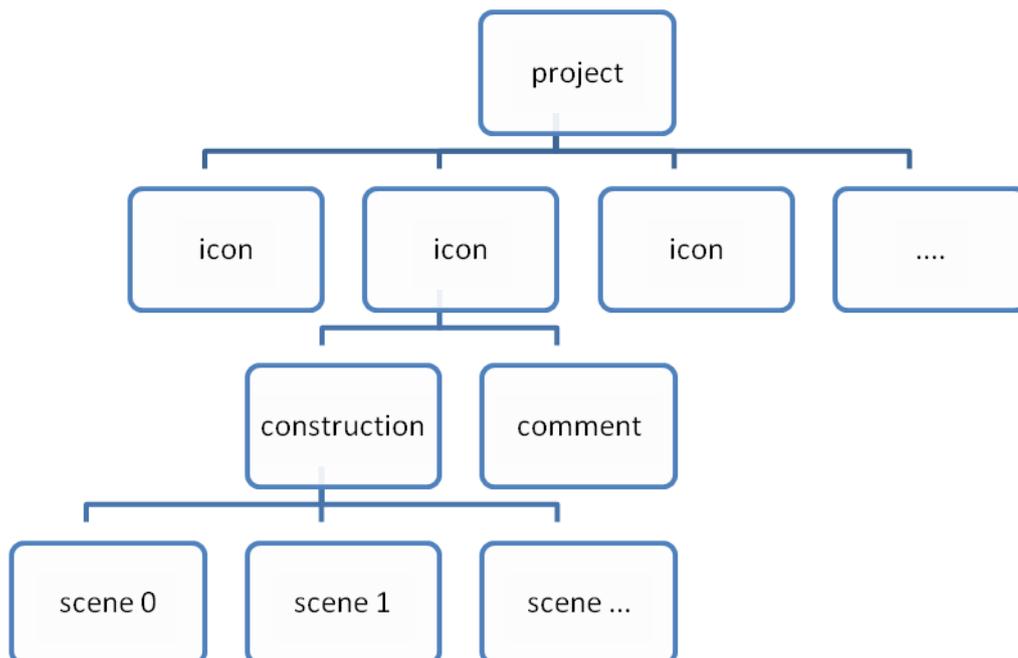


Figure 9

OK Geometry stores projects in files with .pro extension.

1.5 Emphasised, bleached and transparent objects. Auxiliary objects

In addition to the standard attributes for displaying geometric objects (colour, type of lines, width of lines, size of points, fill state), OK Geometry takes into account three additional attributes when displaying them. Objects in icons can be emphasised, bleached or made transparent. The degree of emphasis and bleaching can be dynamically adjusted while visualising a construction in the construction view or icons in the project view.

Emphasised objects are displayed with variable width and possibly by flashing. Emphasised object can be dynamically emphasised as shown in the case of circle and triangle in Figure 10.

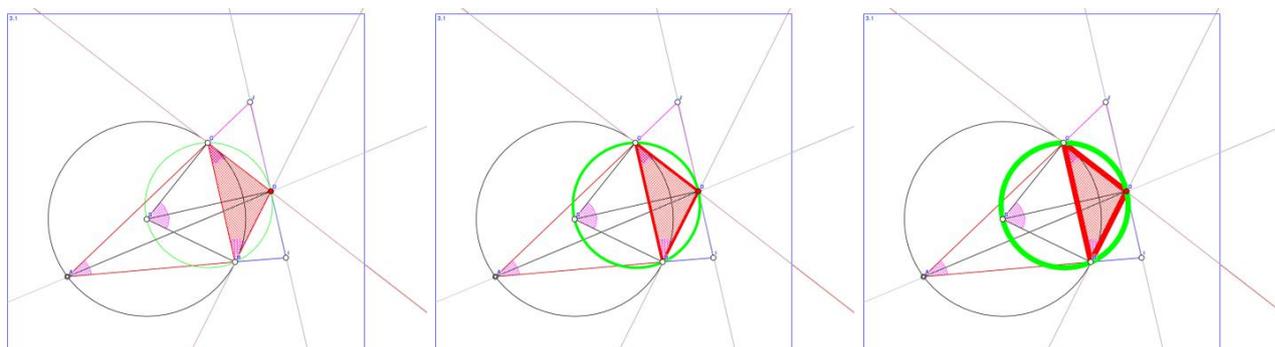


Figure 10

In construction view, you can dynamically control the degree of emphasising with **Shift+Up/Down** keys, or with **Shift+Mouse wheel**. In project view you can control the degree of emphasising with a slider on the bottom left part of the screen.

Bleached objects are displayed with variable level of fading. Bleached objects can be dynamically faded as shown in some angles and triangles on Figure 11.

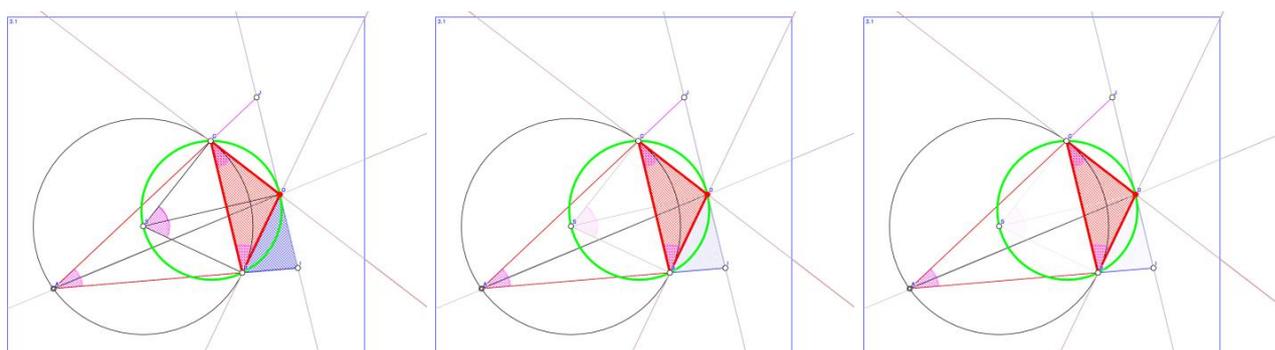


Figure 11

In a construction it useful to bleach objects that are important but may cause unwanted confusion. This way they become less visible. In construction view you can dynamically set the degree of bleaching with **Alt+Up/Down** keys or with **Alt + Mouse wheel**. In project view you can control the bleaching level with a slider on the bottom left part of the screen.

Transparent objects are meant to be invisible. By default, transparent objects are coloured yellow In construction view and they are invisible in project view. There is an important difference between

hidden and transparent objects: transparent objects are considered in observation while hidden objects are ignored. Transparent objects are mostly used in scenes, i.e. an object can be displayed in some scenes and not displayed in others.

In construction view you can consider or ignore the emphasising, bleaching, or transparent attribute (see Section 4.1.7).

Emphasising or bleaching objects and making them transparent is often used in connection to scenes. Recall that a construction can have multiple visual representations, referred to as scenes. An object may be transparent in some scenes and visible in other scenes of the same construction. The same is true for the bleaching and emphasising attributes. See Section 4.1.8 for more information about scenes.

There are also two types of auxiliary objects in OK Geometry. The first type is the **text object** (plain text in the drawing), which can be used to clarify a construction or for other purposes. The second is the **marker object**, which consists of a symbol (①, *, ?, etc.) and text that is only visible when the mouse pointer is positioned on the symbol (Figure 12). Markers are used for different types of comments that should be selectively visible.

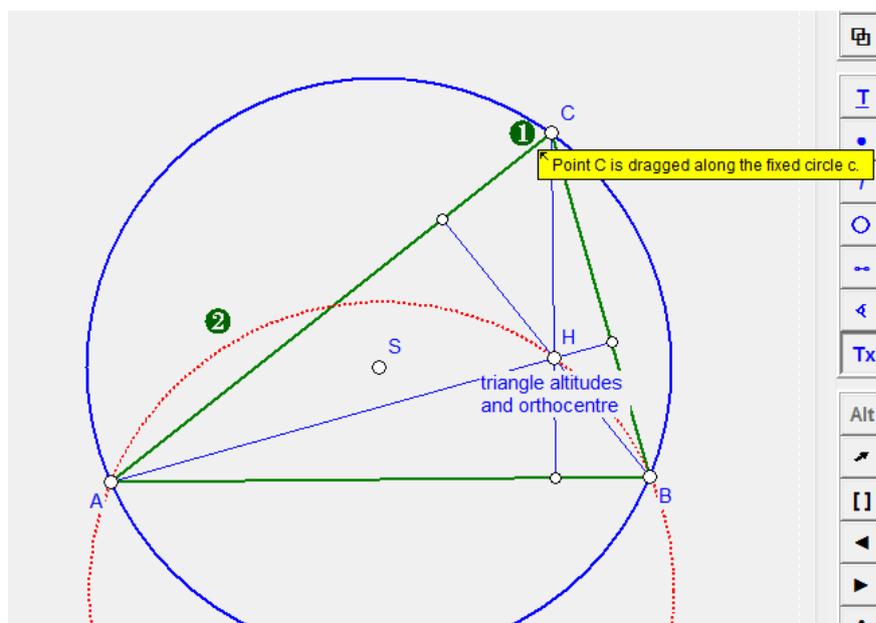


Figure 12

1.6 The interface of OK Geometry

On the top of the window there is a **menu line** with sets of commands (File, Configure, Commands, and Help). Below the menu line there is the **main menu bar** with buttons for the basic commands. Still below the main menu bar is the **information pane**, which consists of **several sections**. The sections can be minimised, expanded or fully expanded – to change their state click or right-click the button (with up/down oriented triangle) on the left of the title of the section. Finally, on the right there is a

visualisation pane. The visualisation pane displays a geometric construction (Figure 13)⁴ or the icons of a project. The details of the interface in either case differ somewhat, but are rather self evident.

There are several ways to access **help** as you work in OK Geometry. Some commands with help information are located under the rightmost menu item. Additional help information for entries is often available as self-appearing balloons or by **right-clicking on object or entry**. During the **Sketch** operation there is also a **Help** section for editing commands, the section can be turned on or off.

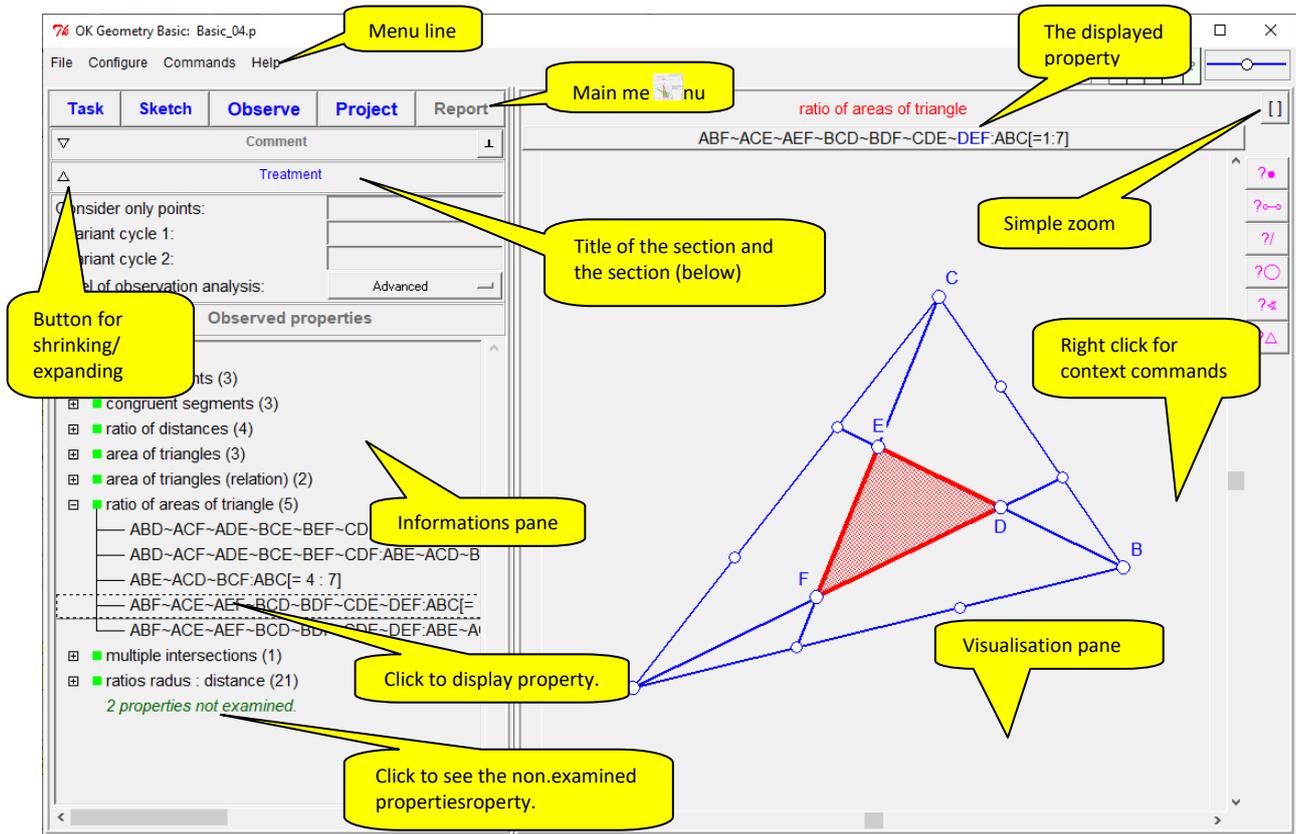


Figure 13

⁴ OKExamples\Basic_04.p

2 Introductory examples

2.1 First example. Detecting properties

We illustrate how to detect geometric properties with Ok Geometry using a very simple situation: Given is a triangle ABC and a point on each of its sides: point D on BC, point E on CA, point F on AB. We will analyse this configuration.

Although it is possible to draw constructions in OK Geometry, in this introductory example we will start with construction files created with a dynamic geometry software Cabri, Cabri II, Cabri Express, GeoGebra, C.a.R. (Compass and Ruler), Sketchometry, Cinderella, or JGEX (see Figure 14). You can find the corresponding files (Example1.fig or Example1.ggb, Example1.jc, Example1.zir or Example1.cdy) among the supplied examples in the directory OKExamples\OKG_Import.

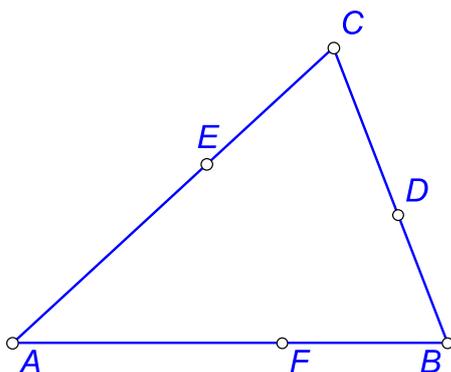


Figure 14

Open OK Geometry in Basic mode and follow these steps:

- Click the button **Task** in the main menu bar and select your file, e.g. Example1.fig in the directory OKExamples\OKG_Import. The construction appears on the right pane.
- Click the button **Observe**. The list of detected properties of the considered construction appears in the left pane in the Properties section (see Figure 15).
- Click on a property you find interesting, e.g. multiple intersections. The property ‘expands’ and instances of the property appear. Click on any instance of the property to visualise it on the right pane. Many of the properties listed are trivial, but you may not have been aware of some non-trivial properties, e.g. three circles passing through a common point (Figure 16) – a fact known as the Miquel theorem.

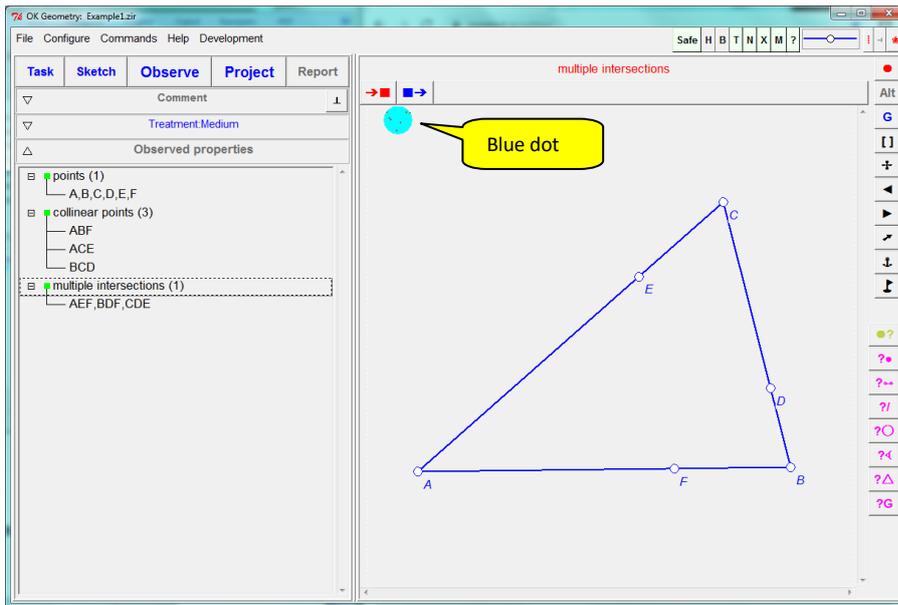


Figure 15

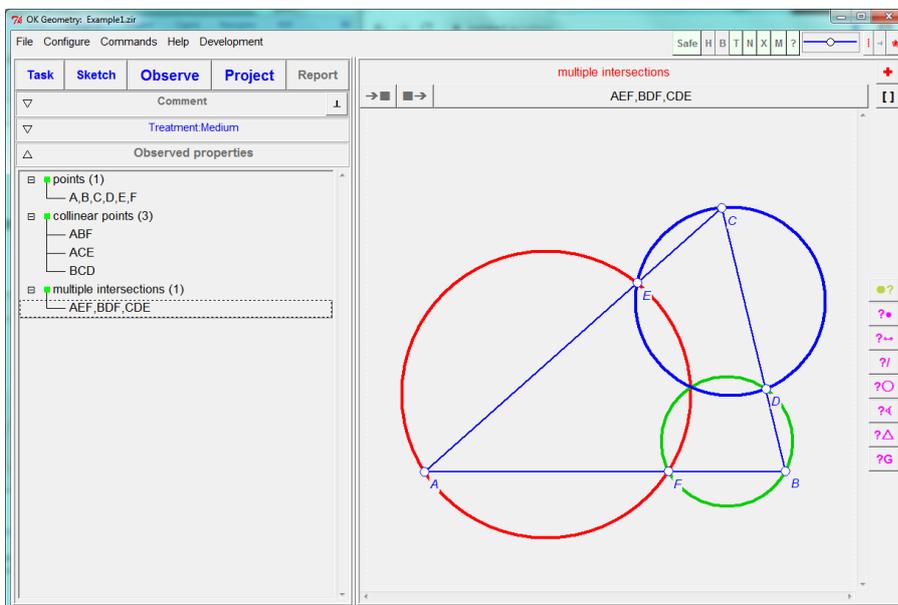


Figure 16

2.2 Second example. Querying objects in a configuration

OK Geometry often detects many properties and many instances of properties. If we are interested only in properties related to a particular object in the configuration, we can use the query commands. Here is a simple example.

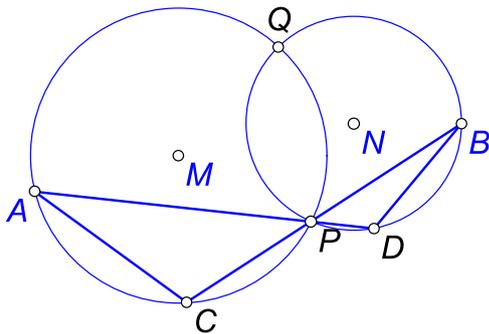


Figure 17

Given are the circles $k(M,A)$ and $k(N,B)$ which intersect at the points P and Q (Figure 17). Let C be the intersection, other than P , of the line BP and circle $k(M,A)$. Let D be the intersection, other than P , of the line AP and the circle $k(N,B)$. On advanced treatment OK Geometry lists more than a hundred instances of properties of this simple configuration, among them 38 congruences of angles. Suppose we want to find information about a particular object, for example, the angle between the lines AC and BD .

We start from a construction in some dynamic geometry software (Example4.ggb, Example4.jc, Example4.txt, Example4.zir, Example4.cdy in the directory OKExamples\OKG_Import) and proceed as in the first example.

1. Apply the command **Task** to import the desired dynamic construction.
2. Apply the command **Observe** to generate a list of observed properties. An extensive list of properties and instances of properties appears.

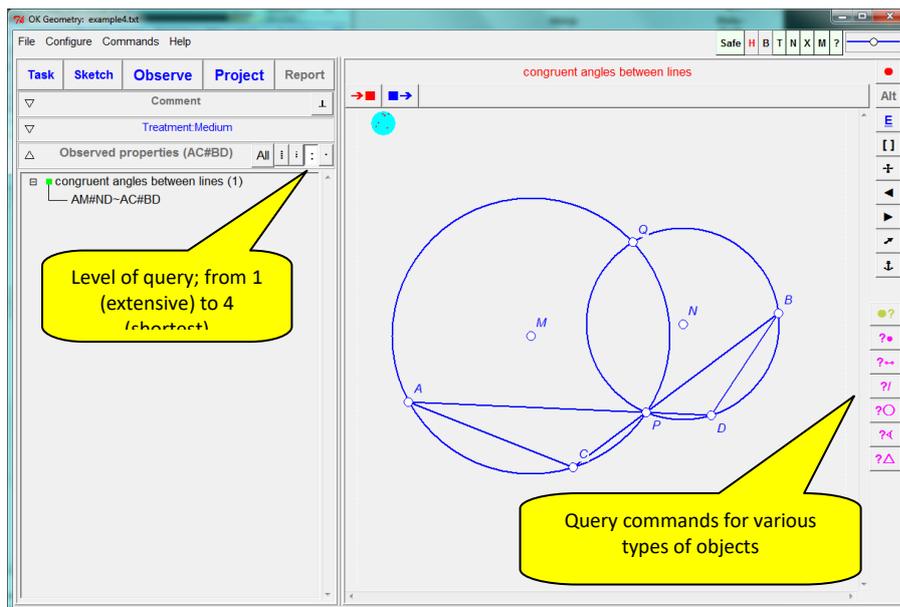


Figure 18

3. On the right taskbar a column of query buttons appears. Click on *QueryAngle*, then pick the segment AC (click anywhere in on the segment) and the segment BD (Figure 18).
4. Select the query level (Figure 18). A higher level (fewer dots) shows more concise information. In our case 1 or 2 (dots) show just one of the angles congruent to the inspected one (Figure 19), while 3 or 4 dots show all angles congruent to the considered one.

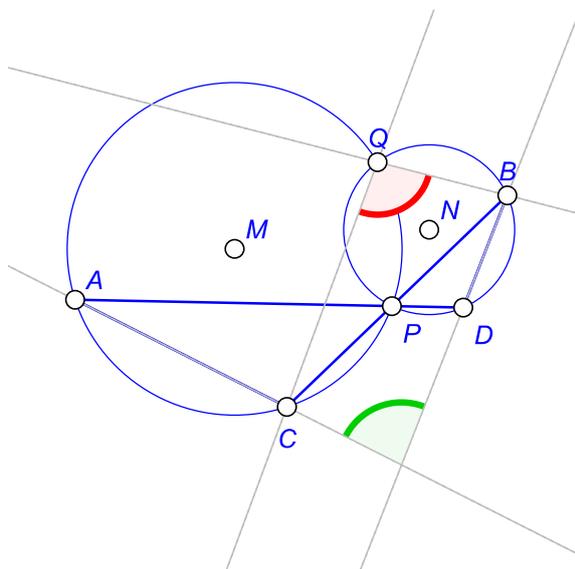


Figure 19

Note. OK Geometry Plus offers a much more sophisticated querying of objects.

2.3 Third example. Working on detected properties

Consider the following task: The diagonals of a trapezium split the trapezium into four triangles. Prove that two of the resulting triangles have the same area.

Also in this example we rely on a dynamic geometry software (Cabri, Cabri Express, Cinderella , C.a.R. (Compass and Ruler), JGEX or GeoGebra). We construct a trapezium (see Figure 20). Note that the vertices should be labelled each with a single letter followed by digits or apostrophhes. We also add to the construction the text of the task. We save the construction in the file Example2.fig (or Example2.ggb or Example2.cdy). You can use the files in the provided examples in the directory OKExamples\OKG_Import.

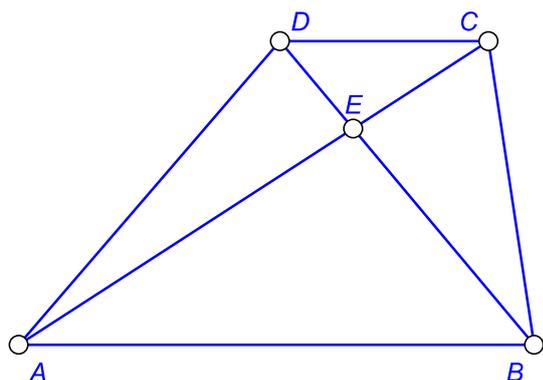


Figure 20

Now open OK Geometry and follow these steps:

Use this button to encapsulate the current construction and text into icon.

Use this button to display any previously made icon.

Figure 21

Use the **Task** command in the main menu bar to import the previously created file that contains the dynamic construction, e.g. Example2.fig in directory OKExamples\OKG_Import (Figure 21). Since we intend to study the properties of this configuration, we declare the imported construction as the basic icon, the Task. To do this, click on the leftmost icon above the visualisation pane and declare the construction as **1 Task** (Figure 21). Apply the command **Observe** on the main menu bar to generate a list of observed properties of the considered construction. Among the listed properties, click on those you consider relevant to your problem, in our case - the areas of the triangles. Three instances of the property will appear. Click on an instance, e.g. $ABC \sim ABD$, to visualise the instance of the property. To visualise a specific object in the relation, e.g. the triangle ABC, click on the name of the object above the drawing pane. (See Figure 22.)

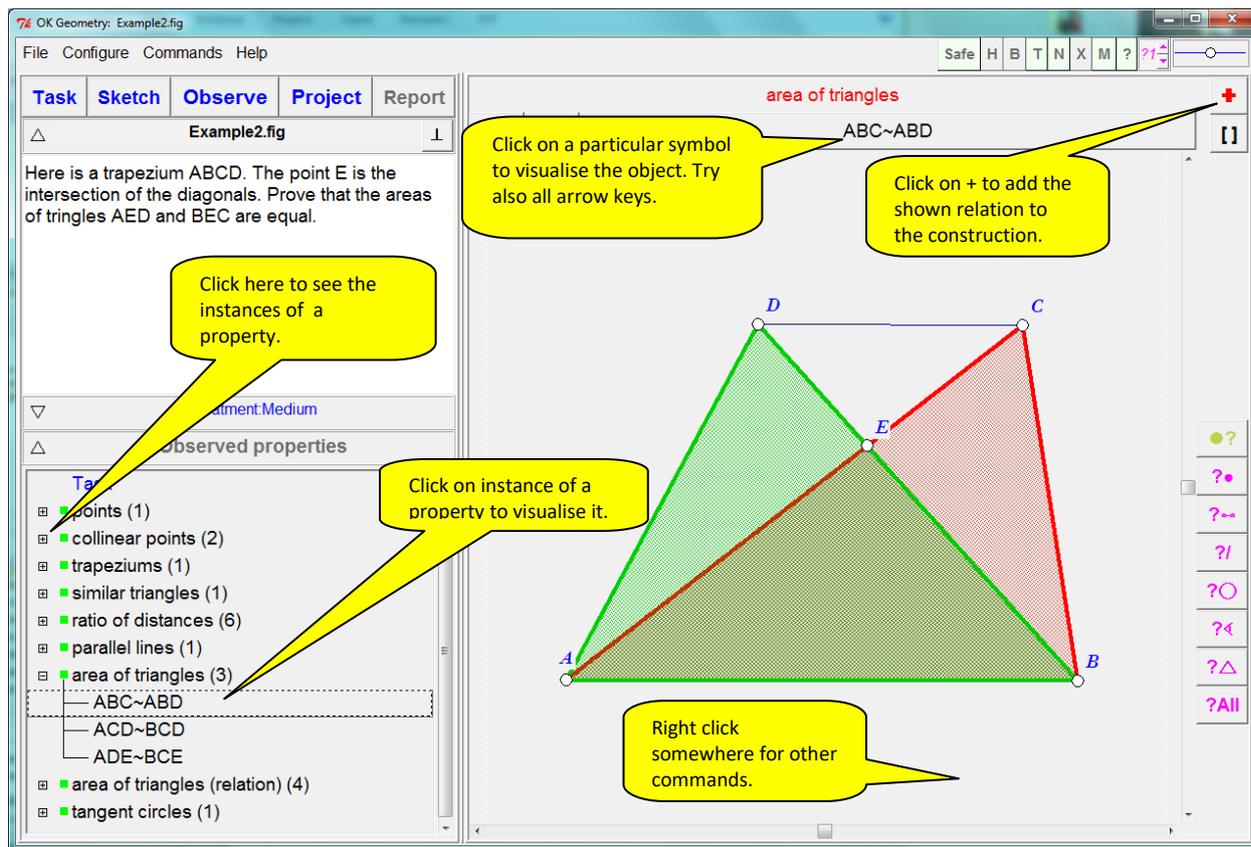


Figure 22

Declare as icons the relations that you think are relevant for further consideration. For example: triangles AED and BEC have the same area, and the triangles ABC and ABD also have the same areas as well. This is clearly related to the fact that in a trapezium the bases AB and CD are parallel. To add the relation to the current construction click on the red '+' icon above the visualisation pane (Figure 21). Usually the obtained construction requires some simple modifications (see Figure 23):

- select a different angle between the two lines using the **anchor button** repeatedly;
- hide objects;
- delete objects;
- modify the colour or style of objects;
- emphasise, bleach or make object transparent.

It is not advisable to drag free points or to re-label the points of the configuration since the icons are (probably) used in relation to the basic task (Task icon).

When the presentation of the relation is as desired, declare it as a **New** icon (or overwrite a previous one) with the **→■** button.

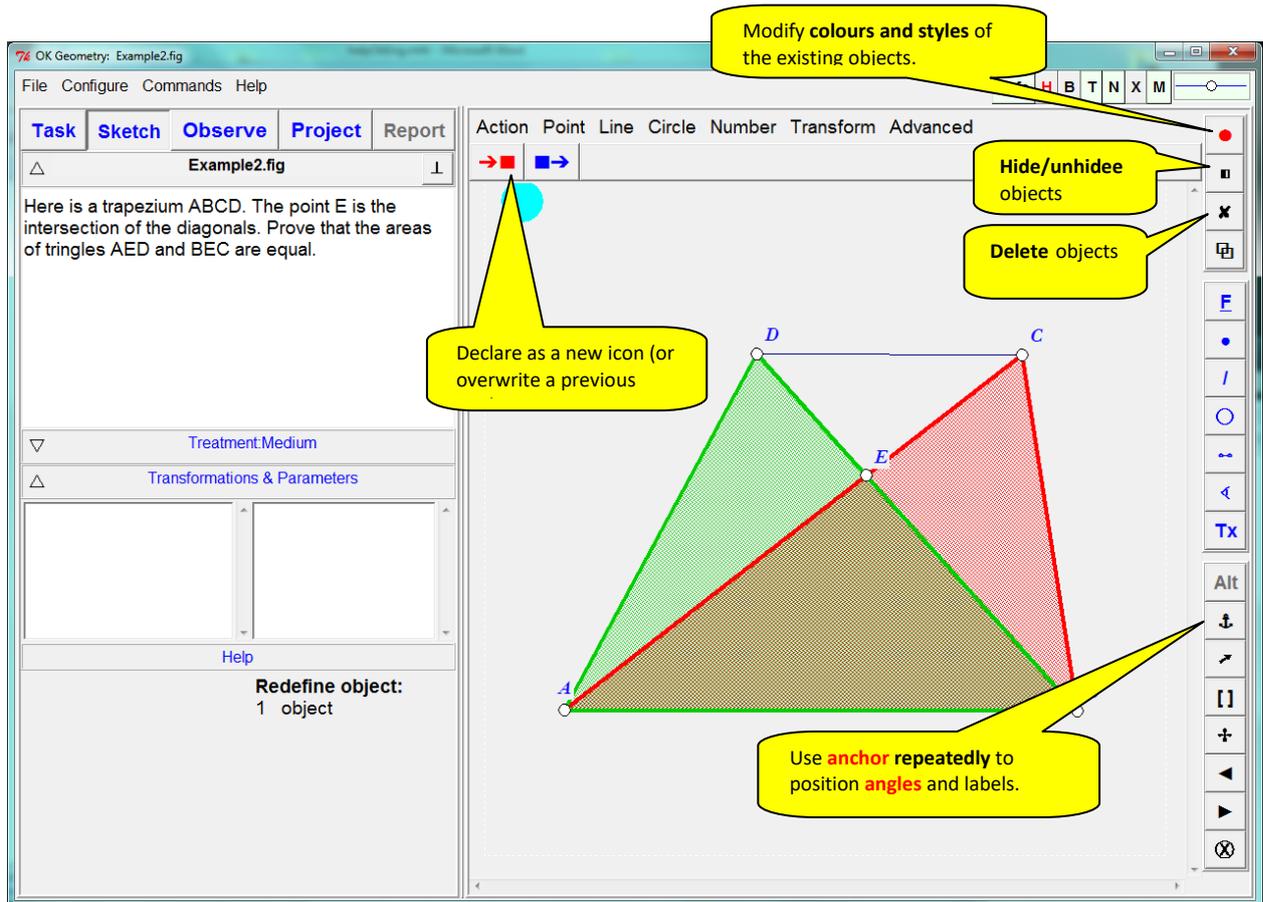


Figure 23

Fill in the form that appears (Figure 24) (you can also write and edit comments later). Note that a paragraph in the comment can be declared as follows: definition/description (D), claim/hypothesis (H), argumentation (A), comment, and formula. Figure 24 shows a commented form for the relation that triangles ABD and ABC have the same area. Click OK to turn the relation into an icon.

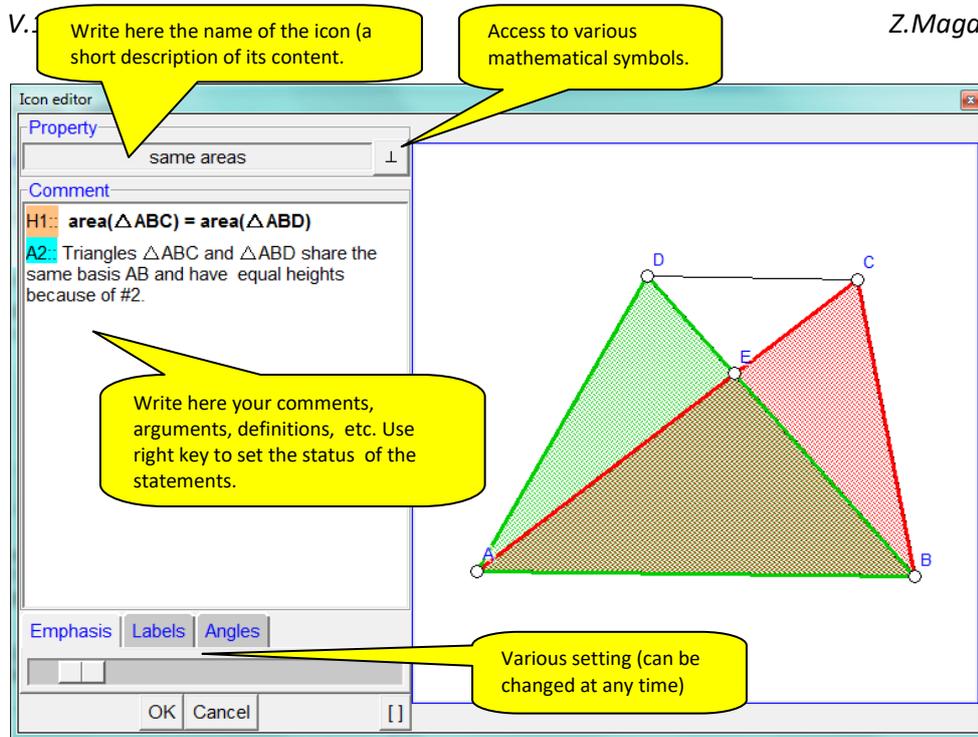


Figure 24

Once you have selected the relevant instances of properties⁵, you can proceed to organise them, to documenting them, or even to build up a proof of a claim. In the main task bar, click the **Project** button (Figure 25). On the left pane you will see the previously selected instances of properties, while on the right pane you will see icons representing the relations. You can move around the icons and change the sequential number of icons. If you want to change the comment to of an icon, simply double click it.

⁵ OKExamples\OKG_Basic\Trapezium1.pro

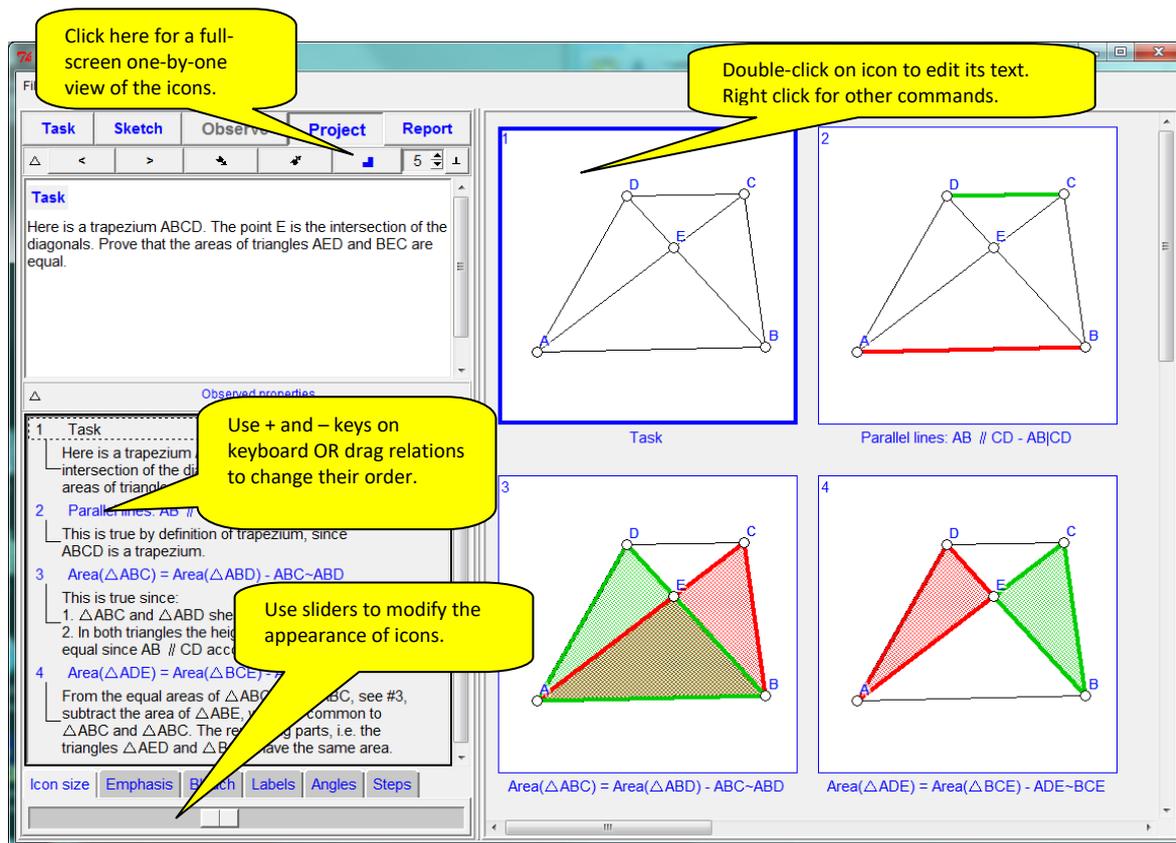


Figure 25

To change the sequential number of the emphasised icon either a) use the + and – keys on the keyboard or b) right-click on the icon and apply the command *Set icon number*. Other attributes of the icons (size of icons, size of labels, emphasis and bleaching effect, size of angle notation) can be changed with sliders (bottom left of the display).

The icons of a project⁶ can be displayed all at once (Figure 25) or individually (Figure 26). To present the icons one-individually, click the  button. Then use the presentation buttons to move between icons. Note: the proof can be organised in a linear fashion or in a tree-like fashion. In the latter case it is possible to specify the level of details of the proof and navigate through the steps of the proof.

⁶ OKExamples\OKG_Basic\Trapezium2.pro

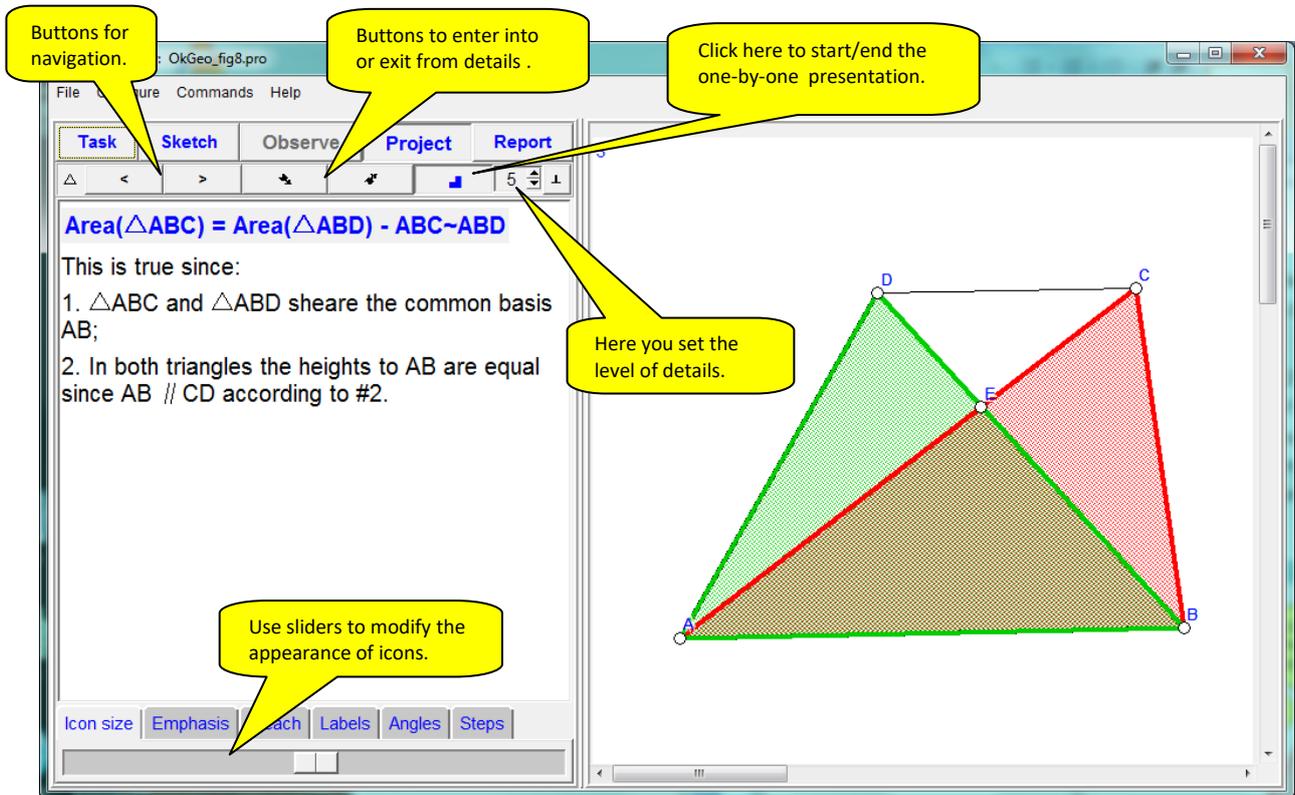


Figure 26

Finally, a report can be generated on the selected properties and the added comments. The report can be saved in .htm, .doc or .tex format. Functionally, the report can be either a memo of the found relations or a classic two column proof of a statement (as built by the user). To create a report, click **Report** on the main tool bar and follow the instructions. The form that appears will ask you to enter some data (Figure 27), the name of the report file, and then to fill out a form like the one below:

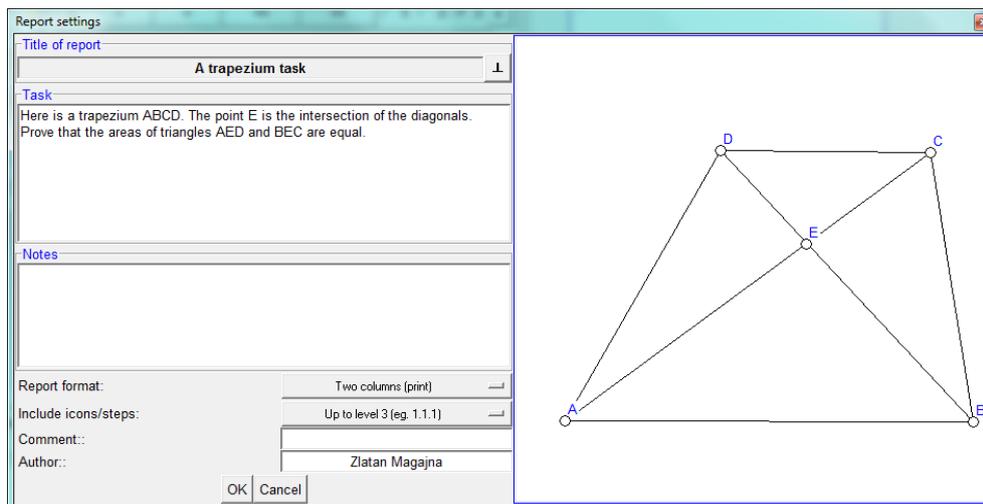


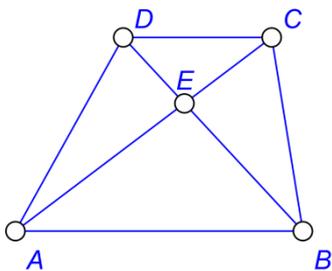
Figure 27

Here is how a report might look like⁷:

Author:Zlatan Magajna (Fri May 6 22:08:02 2022)

Source: OKExamples\ Trapezium2.pro

A trapezium task

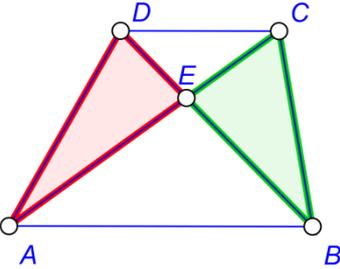


1 Task

Here is a trapezium ABCD. The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.

	<h3>2 parallel lines</h3> <p>Claim 1 $AB \parallel CD$</p>	<p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p>
	<h3>3 same area</h3> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p>	<p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p>

⁷ OKExamples\OKG_Basic\Trapezium2.doc

	<p>4 same area</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p>	<p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p>
---	---	--

2.4 Creating a proof-like task project

Even very simple configurations can have interesting and even unexpected properties. You can use OK Geometry to keep track the configuration and of the found properties. For a given configuration, you can include the configuration itself as well as selected observed properties into a project. The process ‘is described in the previous example (Figure 23, Figure 24, Figure 25). Note, that the iconised properties often require some simple editing (see Section 4.3.1).

You can add iconised properties to a project just to document them. But you can select those properties that can be properly organised into a proof-task. Then you present the icons (in the print form or otherwise) to students either in an ordered or random sequence. The students’ task is to provide arguments and build up a proof with the help of the iconised properties.

Here is an example of a proof-task.

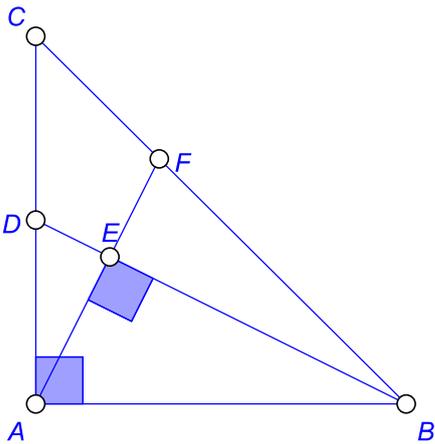
	<p>Given:</p> <ul style="list-style-type: none"> $\triangle ABC$ is an isosceles right triangle. D is the midpoint of AC. E is the orthogonal projection of A onto BD. F is the intersection of AE and BC. <p>To prove:</p> <ul style="list-style-type: none"> $\triangle ABF$ is similar to $\triangle DCF$ <p>Your task is to figure out the proof and to fill in the details.</p>
---	--

Figure 28

A teacher selects and collects the properties to be used in the proof in a project (Figure 29)⁸. Students are asked to solve the proof-task using the icons in electronic or paper form, in correct or random order, with explicit properties or without them.

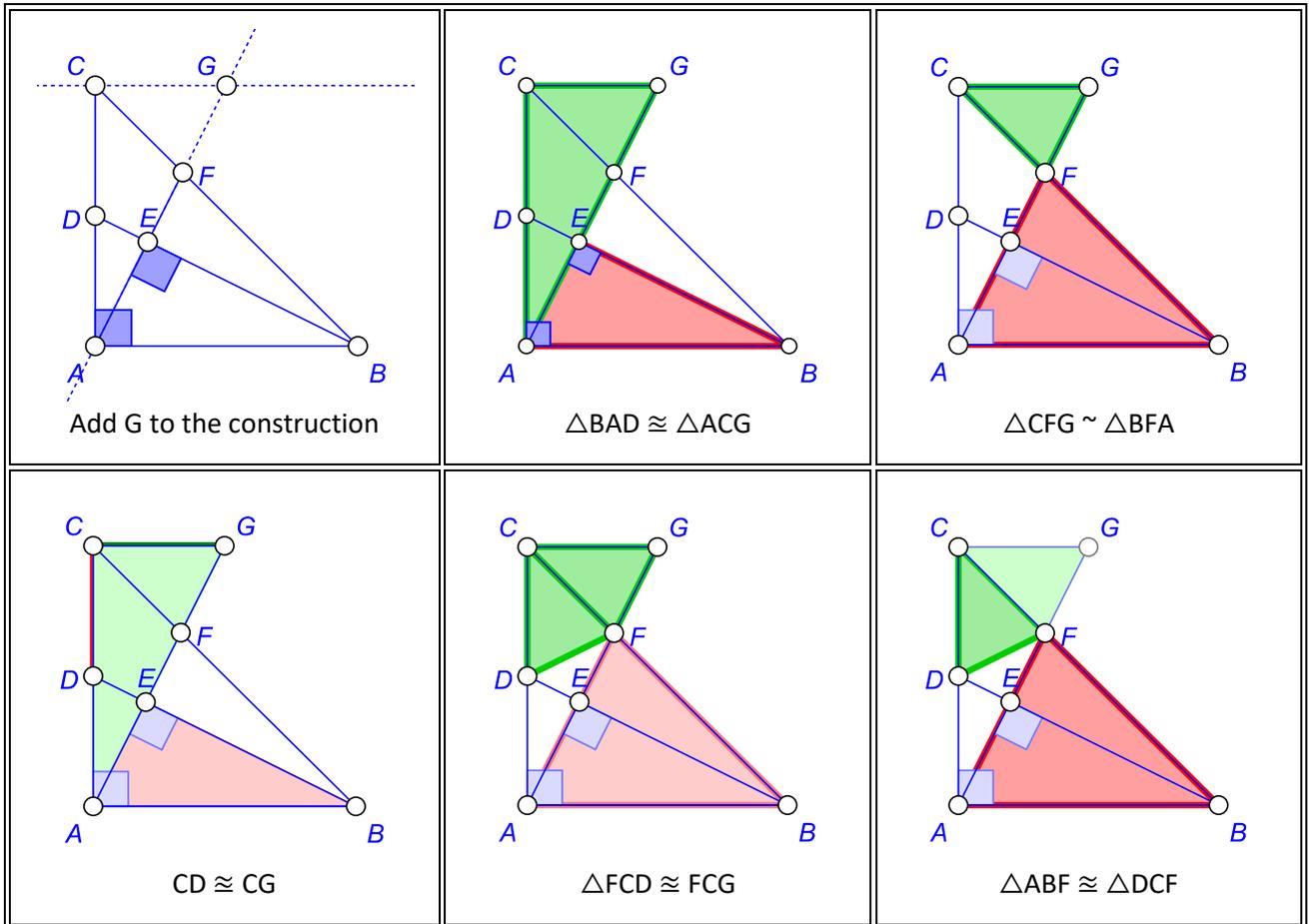


Figure 29

⁸ OKExamples\OKG_Basic\Proof_example.pro

3 Working in OK Geometry

When using OK Geometry, the workflow consists of three steps. It is possible to return to a previous step at any time. The steps are:

Step 1. Obtain a dynamic geometric construction. A construction can be imported from dynamic geometry software or created with the Sketch Editor, a module of OK Geometry. Detailed instructions for the Sketch Editor of OK Geometry can be found in a separate document, the Sketch Editor Reference manual.

Step 2. Analyse the properties of a dynamic construction. Analysing a dynamic construction is very simple. Just click the **Observe** button on the main menu bar and examine the listed properties. In-depth observation of objects and observation of algebraic relations are described in the OK Geometry Plus manual.

Step 3 (optional). Document the observations. Manage a project. Constructions and visualisations of observed properties (see Figure 22) can be collected as icons in a project. The iconised constructions and observations can be edited also later (using the **Project** command in the main menu bar) and documented as a report, see the **Report** command in the main menu bar. For more details see Sections 4.3.4 and 4.3.7.

OK Geometry is often used in a simple way: You simply create or import a geometric construction and use OK Geometry to obtain a list of properties to be studied. It is also easy to produce a report, if one chooses to do so. The following tables describe simple ways to perform steps and also provide a detailed list of specific operations (with hyperlinks).

3.1 How to obtain a construction to be analysed

Purpose	Easy way	Specific operations
<p>Import a construction from Cabri, Cabri Express, GeoGebra, Cinderella, JGEX, Sketchometry or OK Geometry.</p> <p>Edit an existing construction or create a new one in OK Geometry. (See the manual for OK Geometry Sketch Editor.)</p>	<p>Press the button Task in the Main menu bar and import the construction.</p> <p>Press the Sketch button to edit or create constructions within OK Geometry. (See the manual for OK Geometry Sketch Editor.)</p>	<p>Preview of constructions from various dynamic geometry systems</p> <p>Importing constructions from Cabri, Cabri Express, Geometry, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), JGEX, Sketchometry</p> <p>Creating or editing constructions in OK Geometry</p> <p>Labelling points</p> <p>Managing text next to the construction</p> <p>Display construction steps</p>

		Display scenes of the construction Display emphasis, bleaching, and transparency of objects Dyanamically control the degree of bleaching and emphasising objects Exporting figures Saving and reading constructions and projects Recovering the work
<p>Create and edit families of constructions. (Available in OK Geometry Plus. See the accompanying manual.)</p>		

3.2 How to observe and analyse a construction

Purpose	Easy way	Specific operations
Obtain a list of properties of the construction.	Press the button Observe in the main menu bar.	Setting the points to be considered in the analysis Setting the level of analysis Taking into account the symmetries of geometric constructions Blue dot. Perturbation radius Detecting properties Visualising a relation Querying specific objects
Perform an in-depth query of objects in a configuration. (Available in OK Geometry Plus. See the accompanying manual.)		
Perform a triangle analysis , i.e., an analysis with respect to the reference triangle. (Available in OK Geometry Plus. See the accompanying manual.)		

<p>Observe formulae, i.e., observe algebraic relations between geometric quantities. (Available in OK Geometry Plus. See the accompanying manual.)</p>		
---	--	--

3.3 How to document observations and manage projects

Purpose	Easy way	Specific operations
<p>Select and comment relevant properties.</p>	<p>Click the relation to visualise it.</p> <p>Edit, comment and iconise selected relations</p>	<p>Adding displayed objects (showing a relation) to the construction</p> <p>Creating an icon</p> <p>Comment an icon in project view</p> <p>Images and formulae in comments</p>
<p>Organise icons into a document.</p>	<p>Press the Project button to visualise the icons. Organise the icons by dragging the icons. Double click the icon to edit its comment. Right click the properties or icons for additional operations.</p>	<p>Organising icons</p> <p>Commenting icons</p> <p>Editing icon’s construction</p> <p>Conforming icons</p> <p>Presenting individual icons and scenes</p> <p>Saving the project</p>
<p>Create a report.</p>	<p>Press the Report button and follow the instructions.</p>	<p>Making a report</p>

3.4 Configuring OK Geometry

Configuring OK Geometry		Configuring OK Geometry
Shortcuts		Shortcuts

4 Details of the procedures

4.1 Geometric constructions

4.1.1 Importing constructions

OK Geometry imports dynamic constructions from the following software:

Software program	Imported file format (extension)
Cabri Geometry (Cabri Geometry, Cabri Geometry II, Cabri Geometry II Plus)	.fig
Cabri Express	.clmx
GeoGebra	.ggb
Cinderella	.cdy
C.a.R. (Compass and Ruler) or Z.u.L.	.zir
Sketchometry	.jc
JGEX (Java Geometry Expert)	.txt
OK Geometry	.p; .pro

To import a construction, click the **Task** button in the main menu bar and select the file you want to import. You can access to this command also from menu *File/Open construction* or from menu *Commands/Task*. OK Geometry imports geometric objects, some visual attributes of the objects and, obviously, the geometric construction for the analysis.

When creating constructions to be imported to OK Geometry in order to be observed and analysed, we advise to consider the following guidelines for labelling the points of the construction:

- **Point labels** should consist of one or more letters, possibly followed by one or more digits and apostrophes. Here are some examples of allowed labels: A, B, b, P, c', D'', E13, t2', Ab. Labels like E13 or Abc are sometimes displayed as E₁₃ and (Abc). For example, the vertices of the triangle $\Delta E4(Ad)P$ are: E4, Ad, P. Labels are case sensitive. Lower case x,y,z cannot be used as labels. Different points cannot share the same label.
- **A reasonable number of labelled points to be considered in a construction is 4-12.** The number of found relations generally rapidly increases with the number of points to be considered in a

construction. An excessive number of points to be considered can lead to an unmanageable collection of properties or to an unclear visual representation of the properties. During the analysis it is possible to exclude some labelled points from consideration (see section 4.2.1 on excluding points from the analysis and 4.1.5 on changing labels).

Here is a detailed information on how various objects are imported into OK Geometry:

OK Geometry imports many types of geometric objects: **points, lines and line segments, vectors, axes, polygons, circles, arcs, loci, geometric measurements, numeric values, calculations, geometric transformations, constructions, and also macros.**

Note that:

Labels of objects other than points are ignored.

Vectors and axes are imported in OK Geometry but are not displayed.

Rays are imported as lines.

Polygons and polylines are imported, but in some file formats mismatches can occur when polygons or polylines are intersected with other objects. The very safe way of intersecting a polygon with another object is to work with separate segments of polygon or polyline.

Properties of objects. Relevant attributes (visibility, fixed objects) are, in general, preserved as well as the colour and visual appearance of objects. Use the command *Configure/General options/Import attributes of objects* to set whether colour and style of object should be (reasonably) preserved when importing objects.

Text. All text entities are accumulated and imported into OK Geometry as the task description.

Magnification. The construction should be made in a reasonable magnification (e.g. between 1:1 to 1:10). Excesses in this respect can lead to observation errors in OK Geometry.

4.1.2 Preview of constructions from various dynamic geometry systems

The command *File/Inspect files* allows the visual preview of various formats of files in a directory (Figure 30). A previewed construction can be accepted (imported into OK Geometry).

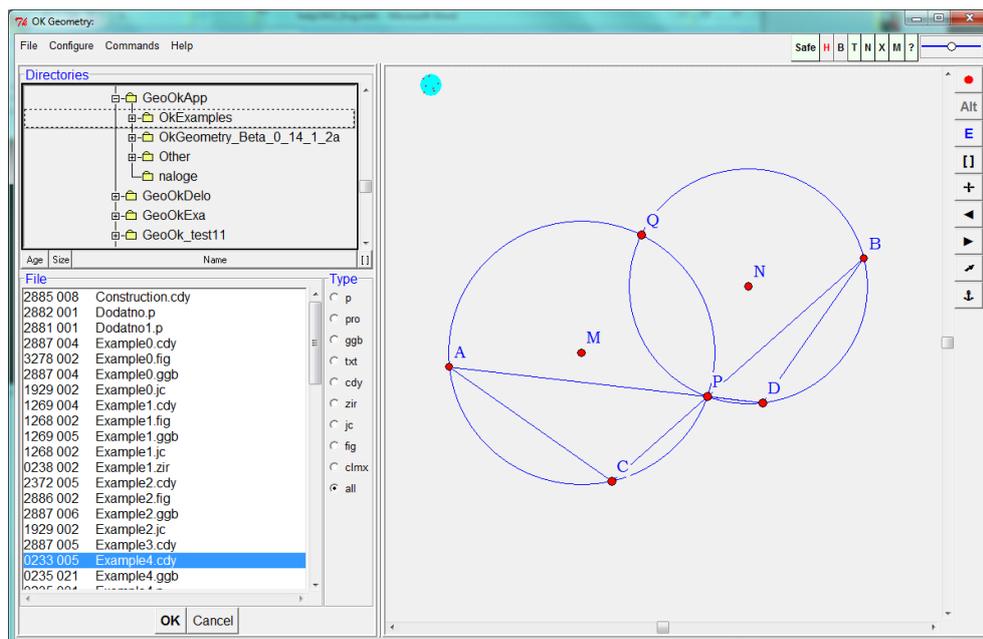


Figure 30

4.1.3 Creating and editing constructions with OK Geometry

You can edit or make new geometric constructions in the Sketch Editor, i.e. in the Sketch mode of OK Geometry. You can access the Sketch Editor by pressing the **Sketch** button in the main menu bar or via the menu *Commands/Sketch*. To exit from the Sketch Editor press any button in the main menu bar.

The Sketch Editor is similar to common editors of dynamic geometry. However, due to specifics of the program, there are some simplifications (e.g. no animation), some specific emphasis, and some specific commands (e.g. commands for complex and implicit geometric constructions). The Sketch Editor of OK Geometry allows to 'construct' a configuration even if one does not know how to construct it as a Euclidean construction. OK Geometry then helps to observe the configuration, and perhaps to discover properties one was not aware of and, consequently, to find ways to create a proper construction and prove its correctness.

Please, refer to OK Sketch Editor Reference for information how to use the editor. There you can also find several examples.

4.1.4 Exporting figures

The displayed construction can be exported at any time as an image in various formats using the *File/Export image* command in the main menu or the context menu (right click on the construction pane). Note that images of constructions are always exported in the exact visualisation mode. A displayed configuration can be exported in three ways:

- Via clipboard for fast copy/pasting of image into another application.
- Via a file in a raster format. You can choose among the commonly used formats (PNG, GIF, JPG, BMP, WMF, TIFF). Such formats are compatible with most commonly used applications.
- Via a file vectorised format: (PS (PostScript), SVG, EMF, PDF). These formats give high precision images in applications that can process them.

Note. Exporting to EMF, PDF format as well as exporting symbols in figures requires a prior installation of Inkscape software⁹, which is freely available.

Exporting strategy. Here is an efficient way for exporting images of constructions.

- Obtain a desired display of the construction. Note that only the indicated square area in the construction pane is exported. Then call the *Export image* command.
- Decide about the desired format/mode of export. If your option is the clipboard or a file in raster format, set the Image quality and image size. In case of a vectorised format, choose appropriate size of figure in the document to be created.
- If necessary, mark the rectangular area to be exported (use the button with a small rectangle on the bottom of the form).
- Use the export button (Clipboard, ToFile(img), ToFile(vect)), select the file and execute the export into the document.

⁹ Inkscape is a free software under GPL license. It is available at <https://inkscape.org/>.

- Often the export result is as expected: The labels are too small or too big, the lines too narrow or too wide, etc.. At this stage correct the Label size, Lines width, and Point size in the export form.
- Repeat the steps to obtain the desired result.

Note. Constructions can be also exported as **GeoGebra static files** (and not as constructions). Such files can be imported in GeoGebra, eventually modified there, and exported from GeoGebra to other formats.

4.1.5 Labelling points

Labelled points play an essential role in the analysis with OK Geometry since all lines, circles, conics, angles etc. 'passing through' the displayed labelled points are considered in the analysis. The labels of points can be set at the time of making a construction or just before the analysis in OK Geometry. All points that are potentially important for the analysis should be labelled.

When labelling the points of the construction to be observed, consider the following requirements:

- **Point labels** should consist of one or more letters, possibly followed by one or more digits and apostrophes. Here are some examples of allowed labels: A, B, b, P, c', D'', E13, t2', Ab. Labels like E13 or Abc are sometimes displayed as E_{13} and (Abc). For example, the vertices of the triangle $\triangle E4(Ad)P$ are: E4, Ad, P. Labels are case sensitive. Lower case x,y,z cannot be used as labels. Different points cannot share the same label.
- **A reasonable number of labelled points to be considered in a construction is 4-12.** The number of found relations generally rapidly increases with the number of points to be considered in a construction. An excessive number of points to be considered can lead to an unmanageable collection of properties or to an unclear visual representation of the properties. During the analysis it is possible to exclude some labelled points from consideration (see section 4.2.1 on excluding points from the analysis and 4.1.5 on changing labels).

You can access the commands for labelling points in three ways:

- Use the button for labelling (a button with the current label inside). A letter P (underlined) button asks for a label to be applied. A letter P (not underlined) button labels an unlabelled point as P, and deletes the label if the point is labelled. (See the Sketch Editor Reference for details.)
- Right click anywhere on the visualisation pane. In the contextual menu click the *Labels* sub-command and then choose the appropriate labelling command (see below).
- When in **Sketch** mode – you can also select the appropriate commands in the editor menu (e.g. *Action/Labels* in the leftmost group of commands in the editor menu).

Note: In order to apply a labelling command, first choose the command, then sequentially apply the command to points you want to label, re-label, or un-label.

Note: Any change of labels can cause inconsistencies with the found relations in a previous observation analysis.

Here is a brief description of the commands for labelling points.

Command	Meaning	Comments
Label vertex	Set the label of a selected point	First click the point you want to label or change the label. Then write the desired label of the point. The program does not accept an invalid label. To manually label more points repeat the operation for each point or skip to <i>Auto label</i> .
Auto label	Let the computer choose the label of a selected point. Clear the label of a labelled point.	The point you click on is labelled with the first available letter after the last applied label. If the selected point is already labelled, the label is cleared. To auto-label more points, just click them one by one in the order you want them to be labelled. The label of the first point can be set with the command <i>Label vertex</i> . Note: A quick way to manually set the next label to be used is to press Alt+S .
Label all	Label all unlabelled displayed points	All shown unlabelled points are labelled in the order as they were defined. Existing labels of points are not changed.
Clear label	Delete the label of selected points	The command deletes the label of the point you click on. To delete the labels of more points just click on each of these points.
Clear all labels	Delete all labels	The command deletes the labels of all points. After this operation all points in the construction are unlabelled.
Anchor label/angle/text/fill	Position the label	The positions of the labels can be modified using the Anchor command or the  button (afterwards try also the mouse scroll button). Labels can also dragged around the referred point.

All points that are potentially important for the analysis should be labelled. Points that are clearly irrelevant should not be labelled: this can be achieved by hiding the points or clearing the labels.

4.1.6 Managing text next to the construction

A geometric construction is usually accompanied by a text that describes the construction or the task associated with the construction. Such a text can be written in the dynamic construction (if imported) as

a common text. It can be also written or edited at any time in the “Task” section of the information pane. The text can consist of the common text, but it is also possible – in a certain way – to include symbols, mathematical formulae, and pictorial information.

Inserting symbols in text

To insert mathematical or other symbols in the text use the symbols table available with the  button on the title line (Figure 31).

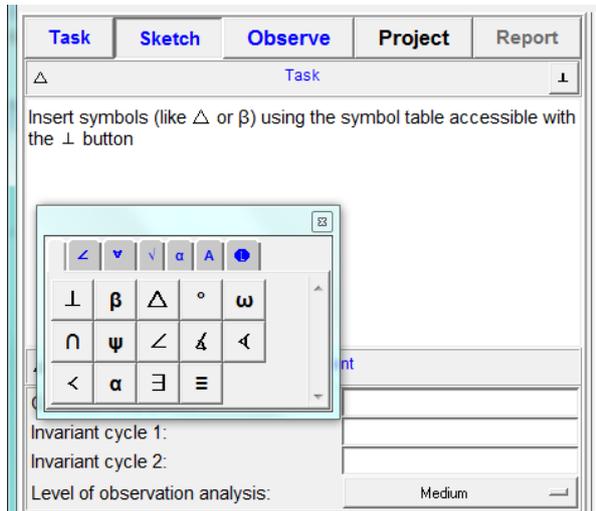


Figure 31

Inserting formulae and pictorial information in text

Images and formulae can be copy-pasted in the text with a right click on the text area (*Image/Paste image* command). To obtain the desired size of the picture in text set the paste size accordingly.

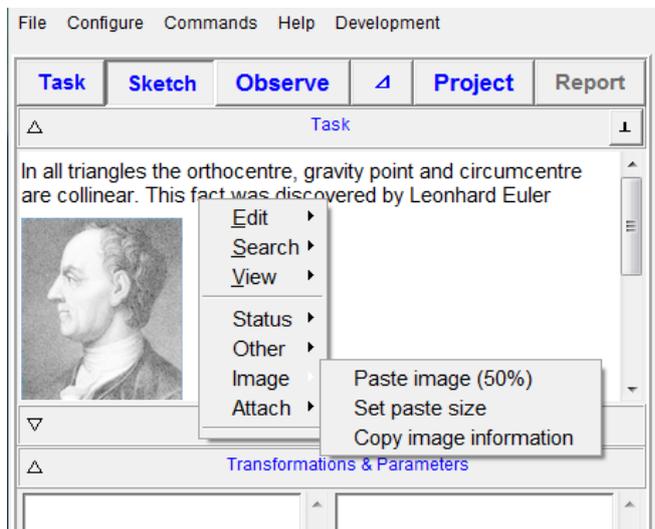


Figure 32

Mathematical formulae can be imported using an appropriate editor. Apart from the pictorial format of the formula, which is imported as image, it is possible to import also the source of the formula. The source has no function in OK Geometry, except that can be retrieved in case one would like to make changes (in the appropriate editor).

Inserting links

Links to files and web pages can be inserted with the Attach command in the context menu (right click on the text area).

4.1.7 Displaying constructions in the construction view

There is a default setting for displaying emphasised, bleached, transparent and hidden objects in the construction view. Normally it is not necessary to modify the default way of displaying constructions, but you can do so if you wish.

In the construction view, you can use the buttons in the upper right part of the display to control emphasis, transparency, bleaching and some other attributes (Figure 7, left, and Figure 33).

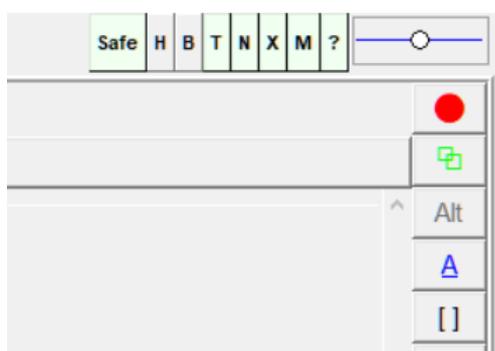


Figure 33

When the **H** button is active, the hidden objects are displayed as if they were not hidden, otherwise they are displayed with a thin red dotted line.

When the **B** button is active, the bleached objects are displayed as if they were not bleached.

When the **T** button is active, the texts in the drawing are visible. Note that this does not affect the visibility of points' labels.

When the **N** button is active, the transparent objects are visible (coloured in yellow), otherwise they are not displayed.

When the **X** button is active, the auxiliary lines (related to angles) are visible, otherwise they are not.

When the **M** button is active, the markers are visible, otherwise they are not visible.

When the **?** button is active, the unknown objects of a construction are highlighted. (See section Advanced observation in OK Geometry Plus reference.)

You can interactively change the level of emphasis (of the emphasised objects) by scrolling with the middle mouse button while pressing the **Caps** key.

You can interactively change the level of bleaching (of the bleached objects) by scrolling with the middle mouse button while pressing the **Alt** key.

4.1.8 Scenes

Scenes are visual representations of constructions. A construction always has a default representation called the **ground scene**, but it can have other scenes. All scenes of a construction are based on a common construction, the scenes differ only in the collections of objects that are emphasised, made transparent or bleached. Figure 35 shows 6 scenes of the same construction. As you can see, an object may be emphasised in one scene, invisible in another, and bleached in yet another. (The scenes illustrate a proof that the baselines of the altitudes of a triangle meet at a common point.)

By default, you will see only the ground scene in **construction view**. By default, the transparent objects are coloured yellow (Figure 34, right) and you can control the degree of bleaching and emphasising of objects (see Section 1.5). You can use the **N** button (see Figure 33) to make the transparent objects invisible, and the button **B** (see Figure 33) to ignore the bleaching attribute of objects.

To see the scenes (if there are any besides the ground scene) in the construction view, click on the green

Scenes button  on the right side of screen or use the *Command/Construction steps* command.

The scenes of a construction are created and managed using the **Scenes** command in the Sketch Editor.

In **project view** the scenes are only visible when individual icons are displayed. See Section 4.3.6 for details.

You can use the file *Scenes_example1.p* and *Scenes_example2.p* in the directory *OKExamples\OKG_Basic* to try these commands. The construction contains 6 scenes that illustrate the proof of the existence of the orthocentre of a triangle. Figure 34 shows the construction when the transparent lines are invisible (left) and visible (right). Figure 35 shows the scenes in the construction. As you can see, all scenes contain the same objects, but the visibility, bleaching, and transparency state of each object can vary from scene to scene.

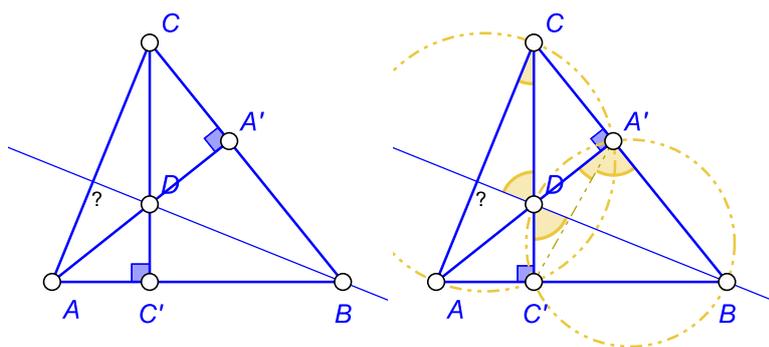


Figure 34

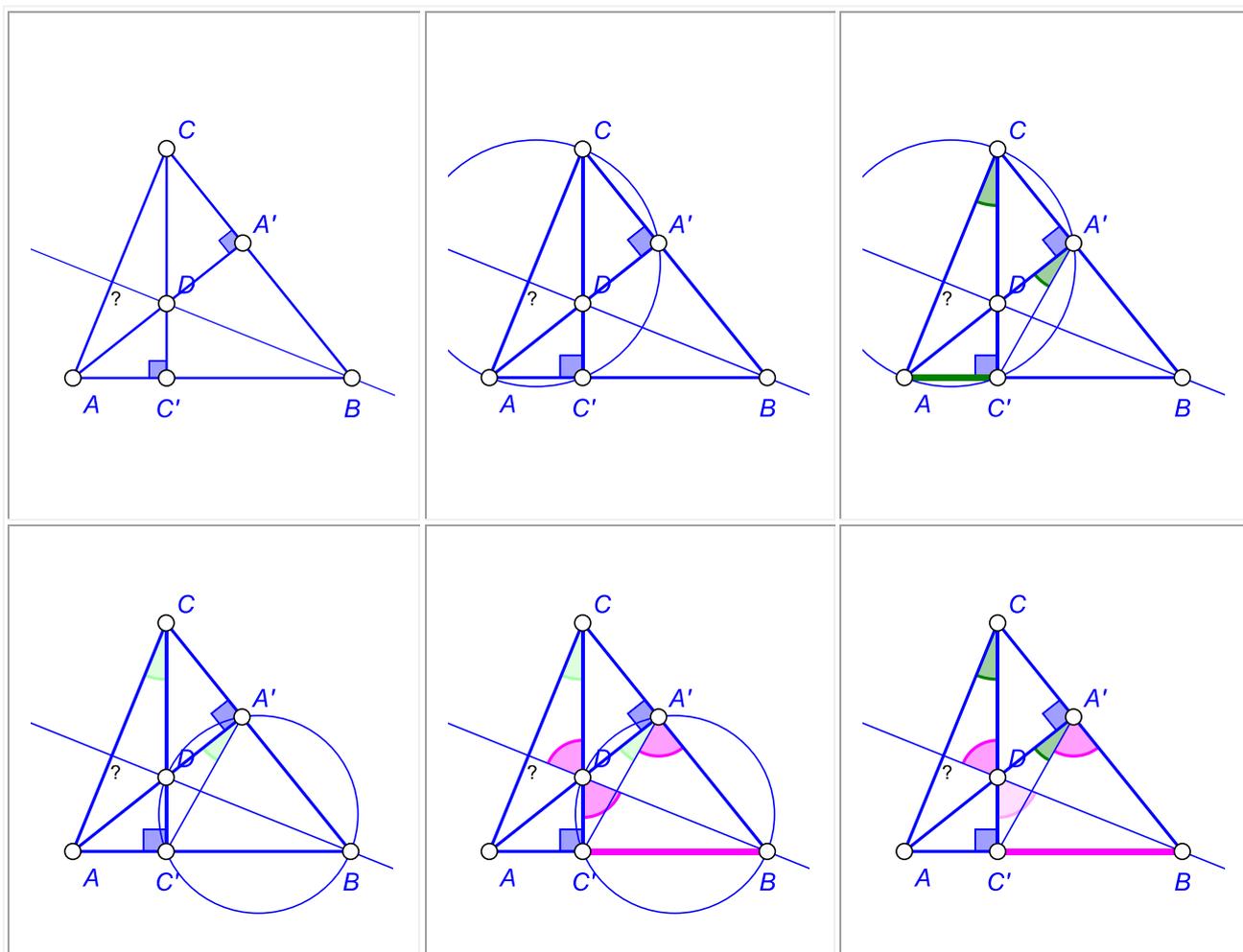


Figure 35

4.1.9 Saving and retrieving constructions and projects

OK Geometry stores data in two types of files. The current construction is saved in file with the **.p** extension, the current project is saved in file with **.pro** extension.

Saving constructions. To save the current construction to a file, use the *File/Save construction* command from the menu. The construction is saved in OK Geometry format. Note that the saved construction also contains the information about the Task section and Treatment section of the information pane but does not contain any information about icons and properties found or considered.

Retrieving constructions. A saved construction can be retrieved with the *File/Open construction* command from the menu or with the Task button.

Saving and retrieving projects. Current projects with commented icons comments and underlying dynamic constructions are saved and retrieved in files with **.pro** extension. To save the current project use *File/Save project* command from the menu. The work that was saved in a **.pro** file can be retrieved using the command *File/Open project* from the menu.

4.1.10 Recovering the current work

Recovering the current work. OK geometry continuously saves the current work. The last saved work can be recovered using the *Configure/Recover last session* from the menu command.

4.2 Observing and analysing constructions

In most cases there is no need to configure the analysis. Simply press the **Observe** button in the main menu bar. Occasionally, however, simple adjustments might be helpful.

4.2.1 Setting the points to be considered in the analysis

Why setting the points for the analysis?

OK Geometry in a construction only takes into account the labelled points (see Sections 4.1.1 or 4.1.5). It is therefore important that all potentially relevant points are labelled. However, an excessive number of labelled points leads to some problems: The list of relations, observed by OK Geometry, can be so extensive that it is difficult to identify the interesting ones among them. Also, the observation process can take a while, and the detection of certain properties can be unreliable. Thus, it is advisable to restrict the analysis to 4-12 points and, eventually, repeat the analysis on various sets of points of the same configuration. For this reason OK Geometry allows to specify which points to consider in the analysis without changing the construction itself.

The points to be considered in the analysis are specified in the **Treatment section** of the information pane. The points to be considered in the analysis should be entered in the entry **Consider only points**. If the entry is empty, all labelled points will be considered in the analysis, otherwise only the declared points will be considered. If there is an exclamation point at the beginning, the specified points will not be included in the analysis.

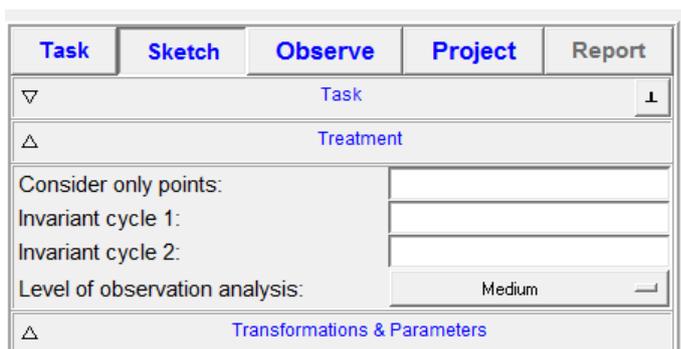


Figure 36

Here are some examples for the entry **Consider only points**:

empty – All labelled points in the construction are considered in the analysis. This is the default assumption.

AB'C1(DG)R – Only points A, B', C₁, DG, and R are considered.

!ADH - All labelled points *except* A, D, and H are considered.

The considered points are saved as a part of the geometric construction. When you retrieve a construction (in OK Geometry format) you retrieve also the information which points to considered in the analysis.

4.2.2 Symmetries in geometric constructions

With this feature we impose an additional structure in the set of detected relations when we explore properties that are invariant under given rearrangements of vertices, so that the results are much more reliable and also easier to manage. It is possible to choose between two types of rearrangements:

Cyclic rearrangements. We use the notation like $ABC \text{ DEF}$ when we look for properties that are preserved if the vertices A,B,C , and D,E,F are cyclically changed to C,A,B , F,D,E or to B,C,A , E,F,D . In other words, we require that the observed properties are invariant under any cyclic permutations of ABC and the corresponding cyclic permutations of D,E,F .

Complete rearrangements. We use the notation like $*ABC \text{ DEF}$ when we look for properties that are preserved if the vertices A,B,C , D,E,F are rearranged to C,A,B , F,D,E or to B,C,A , E,F,D or to B,A,C , E,D,F or to C,B,A , F,E,D or to A,C,B , E,D,F . In other words, we require that the observed properties are invariant under permutations of ABC and the corresponding permutations of D,E,F .

Let us explain this in with some simple examples. (You can use the file *Incircle_example.p* in the directory *OKExamples\OKG_Basic* to try the rearrangements described below.)

Given is a triangle ABC . The inscribed circle (with centre G) of this triangle touches the sides of the triangle in points D , E , and F (see Figure 37).

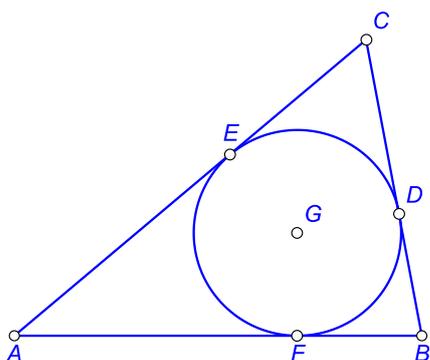


Figure 37

In this configuration there are, for example, 6 right triangles:

AEG, BDG, CEG,
AED, CEG, BEG.

You can easily see that these triangles refer to, basically, the same property (right triangle). We can reduce the 6 instances of this property to just one, for example AEG. All the other instances are

obtained by permutations of the vertices A,B,C and respective permutations of D,E,F. We symbolically describe this invariance by writing *ABC DEF in the entry **Invariant cycle 1** (Figure 38). As a result of the observation, OK Geometry lists 6 right triangles. Note that 5 instances of the right triangles are written in blue colour, indicating that these instances are, basically, ‘repetitions’ of some other instance of property. The blue coloured instances of properties are thus obtained from other instances properties by applying the permutations in the Invariant cycle entry or entries.

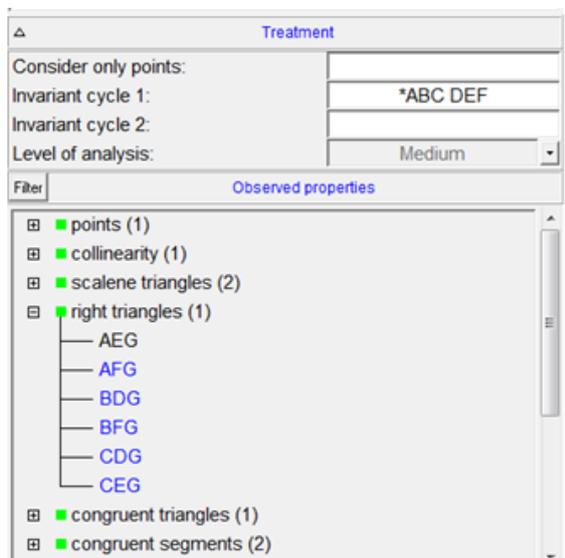


Figure 38

Let us now consider a more sophisticated example: a kite (Figure 39). You can use the file *Kite_example.p* in the directory *OKExamples\OKG_Basic* to try the rearrangements described hereby. The kite has a line of symmetry, thus its properties are preserved if B and D are swapped. Setting the **Invariant cycle 1** entry to BD reduces the number of essential relations. Note that the geometric properties are also preserved when A and C are swapped independently of B and D. Thus setting the **Invariant cycle 2** entry to AC reduces further the number of essential relations.

What happens if the **Invariant cycle 1** entry is set to ABCD and the **Invariant cycle 2** entry is left empty? The property that the triangle ABD is isosceles will not be listed since the triangle BCA (obtained by a cyclic permutation of the vertices) is not isosceles. However, the fact that the line AC is perpendicular to BD will be listed because it is clearly invariant under the cyclic permutations of the vertices A,B,C,D.

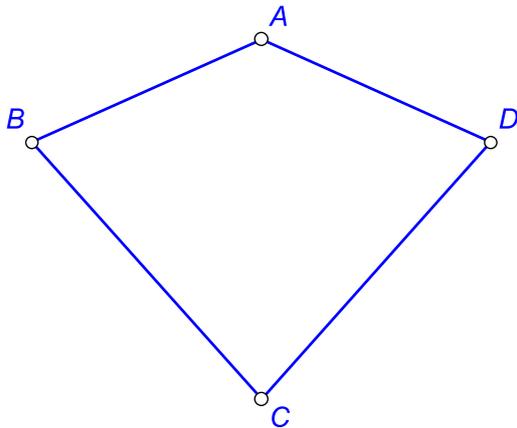


Figure 39

Some additional notes:

- By default, the **Invariant cycles** entries are empty, so OK Geometry does not assume invariance of properties under permutation of vertices.
- The syntax of each entry is evident from the examples above. Note that each entry should consist of one or more cycles of equal length. A given point can occur at most once in each entry.
- The data of the invariant cycles are part of the construction. Thus they are stored together with geometric constructions. When you retrieve a construction (in OK Geometry format .p), you also get its invariant cycles.

4.2.3 Setting the level of analysis

You can choose between these levels of analysis: Simple, Medium, Advanced, and Complete. The default level of expertise is set in the menu command *Configure/General options/Observe* under the entry **Default level of analysis**.

The level of analysis for the current task is set in the Treatment section of the Information pane, as shown in Figure 40.

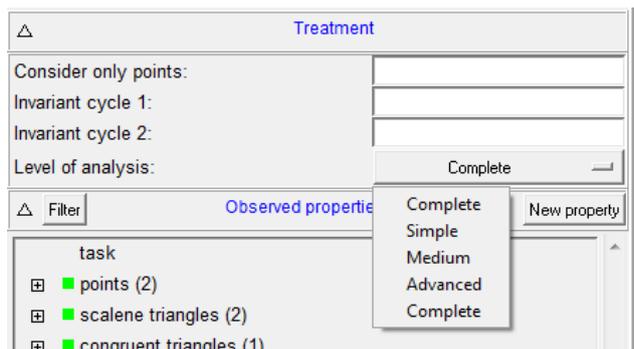


Figure 40

Here is a very short description of the levels of expertise:

Level	Description of the considered properties
Simple	Basic properties of geometric objects, specific shapes (of triangles), congruence
Medium	Simple + ratio of lengths and angles, similarity
Advanced	Medium + advanced ratios of lengths, points on conics, golden ratio, cross-ratio
Complete	Advanced + relations that often give rise to a big number of properties (e.g. pairs of angles with the same sum). The observations at this level can result in a huge number of relations, so this level is rarely used.

Detailed information about which properties are taken into account at a particular level can be obtained from the *Configure/Considered properties* menu command.

The choice of the level of expertise is, of course, a matter of user’s expertise in geometry and of the content of the task. In general, you should choose between Simple, Medium, and Advanced level. The Complete level should be used with caution, since the some properties, specific to this level, sometimes generate a long and unmanageable list of observed relations.

Note. OK Geometry Plus observes many additional properties related to triangle geometry.

4.2.4 The list of found properties

The main feature of OK Geometry is the ability to detect geometric properties in a geometric construction. This is accomplished with the **Observe** button in the main menu bar or with the *Commands/Observe* command in the main menu. The command **Observe** explores whether the objects (determined by the labelled points) in the current geometric construction have certain geometric properties.

The **Observe** command generates a list of properties and instances of properties of the examined geometric configuration. The list is fairly self-explanatory, but some conventions used should be mentioned. Figure 41 shows an example of a list created with the **Observe** command. For each considered property there are 4 possible outcomes:

No instance of the property was found. In this case, the property is not listed. In Figure 41, for example, no right triangles were found, so the property is not listed.

Some instances of the property were found. In this case, the property is listed with an expansion sign (a + in a small square) in front of the property and the number of instances after the property. In Figure 41, for example, OK Geometry has found 4 instances of equilateral triangles. To see the found instances, simply click the expansion sign.

The property was not checked. There are two reasons why OK Geometry does not check a property. There may be so many instances of properties that need to be checked that the computation would take too much time. It is also possible that OK Geometry finds that the results of the observation of the property are too unreliable.

If the number of instances of a property exceeds a threshold (set in *Configure/General options/Observe*), a notice 'x properties not examined' appears at the end of the list (Figure 41, left). Click on the notice to see the affected properties and respective number of instances (Figure 41, right).

There are at least two ways to overcome such a situation:

- The preferred way is to reduce the number of points considered (see 4.2.1).
- You can increase the threshold for the allowed number of instances of properties to be checked (see Section 4.4.2, *Configure/General options/Observe*). Then repeat the **Observe** command.

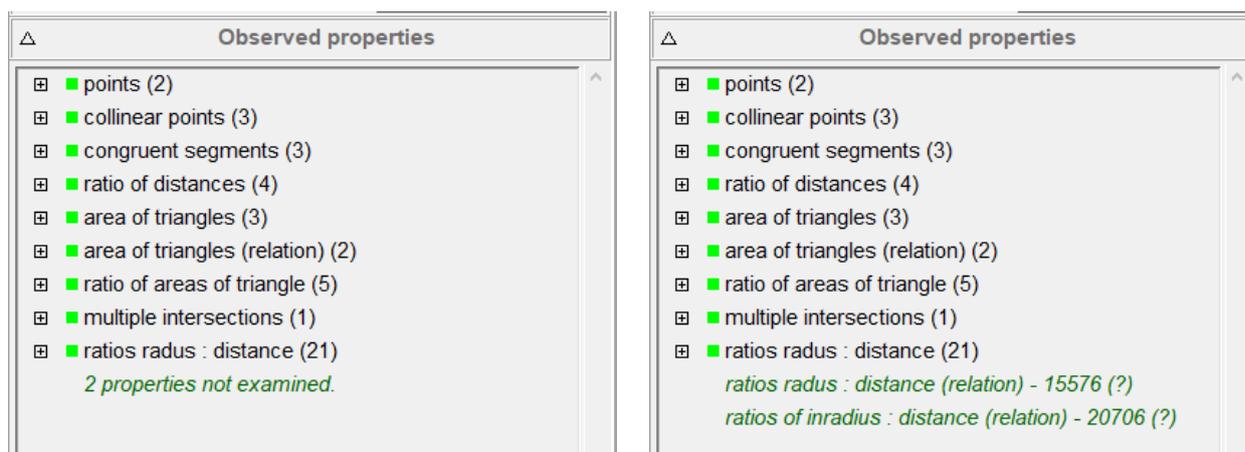


Figure 41

Note the coloured squares in front of the names of the properties. The colour of the square indicates the probability of error of the observation:

green square - the probability is extremely low;

yellow square - the probability low;

orange square - the observed relation is possibly a result of chance;

red square - OK Geometry has detected a suspicious feature in the construction that may call into question the validity of the observation.

The reliability of the results obtained with OK Geometry **is significantly improved**

- if symmetries in the construction are taken into account (see Section 4.2.2) or
- if the number of points considered is reduced or

- if the level of perturbation is increased (see menu command *Configure/General options/Perturbation magnitude*)

On the other hand, the observations are unreliable

- if the construction is based on optimisation or the construction is implicit (in this case also one or two red triangles appear at the upper right corner of the display) or
- if the number of considered points is excessive.

4.2.5 Instances of properties

If a property is preceded by an expansion mark, the instances of the property can be displayed by clicking on the expansion mark. The interpretation of the listed instances of properties is often straightforward, especially if there is its graphic representation in the visualisation pane. For example, in the situation shown in Figure 42 OK Geometry has found three instances of pairs of congruent triangles.

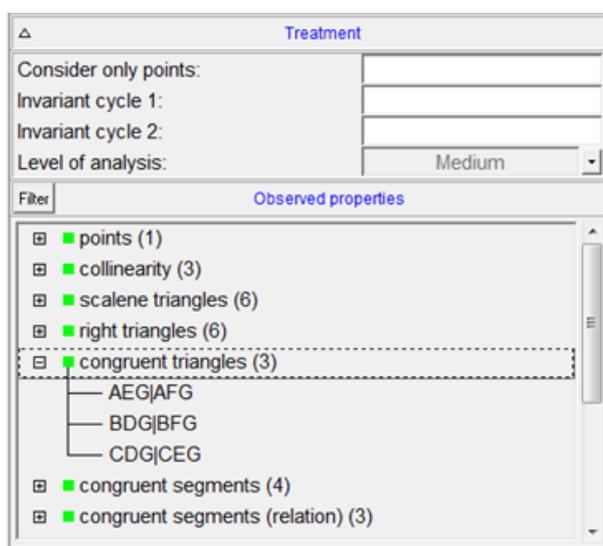


Figure 42

Usually, there is no need to bother about details in symbolic expressions of relations. Nevertheless, here are the basic rules:

- Objects are represented by juxtaposition of points that define them. From the name of the property, we infer the type of the objects involved (e.g., collinearity is clearly about points on lines, similar triangles is clearly about triangles). The objects are then identified by points that define the objects. For example:
- Lines are identified by two or more points on a line (e.g., AC, AGTU). Note that the order of the points **does not** indicate the order of the points on the line.
- Circles are identified by 3 or more points on the circle (e.g., ABC, ABMR). Note that the order of vertices **does not** indicate the position of the points on the circle.
- Triangles are denoted by the three vertices (e.g., ABC, AMP). The order of vertices has **no** geometric meaning.
- Angles between lines are represented as AB#CD (in this case the angle between the line AB and the line CD; here the vertex of the angle need not be part of the construction). Similarly,

$ABCD\#EF$ represents the angle between the line containing the collinear points A,B,C,D and the line EF . <in this case **we also do not distinguish between an angle and its supplementary angle**. More precisely: $ABCD\#EF$ denotes the non-oriented angle (between 0 and 90 degrees) between the lines $ABCD$ and EF .

- Properties refer to objects of the same type (actually, there are a few exceptions to this). Usually, the presented objects have the same property. However they are listed with separators in different hierarchical order. In practice, you need not bother of the meaning of the separator symbols – they are fairly self-evident, especially when we visualise the relation. The meaning of the separators (in order from high precedence to low precedence) is:

- | parallel lines, congruent segments, congruent triangles, etc.
- # angle sign
- ~ same quantity (e.g. angle, area, perimeter), similar triangles
- + sum of quantities (lengths, angles, etc)
- : ratio of quantities, also tangency
- ,
- = same numerical values (e.g. same sum, same ratio, etc.)
- ;

Here are some examples:

Similar triangles: $ABC APD\sim EFG$	In this case the triangles ABC , APD , and EFG are similar, but ABC and APD are even congruent. Note that the order of vertices means nothing, i.e. $ABC APD$ does not mean that AB is congruent to AP .
Sum of lengths: $AP AC+EF=CD+CE BC.$	In this case the segments AP and AC are congruent, and so are the segments CE and BC . Furthermore the lengths $AP+EF$, $AC+EF$, $CD+CE$, $CD+BC$ add to the same sum.
Same angles: $AB CD\#EF GH [=30\text{ deg}].$	In this case OK Geometry has found an angle of 30 degrees. Actually there are two pairs of parallel lines: AB is parallel to CD , and EF is parallel to GH . The angle between the directions of these lines is 30 degrees.

4.2.6 Visualising relations

To visualise a relation, just click on the displayed instance of property in the Information pane (see Figure 22). For simple properties an iconic representation with obvious interpretation appears in the visualisation pane. For complex properties (e.g. when a property involves many objects) you need to visualise smaller groups of involved objects or even single objects. There are two easy ways to achieve this:

- When a property is visualised, note that the property is written also in the visualisation pane, just above the drawn construction. Click with the mouse on an object in the written representation, and the object becomes highlighted. If you click again on the same object then the related groups of object becomes highlighted (see Figure 22).
- A more flexible way of visualising complex relation is via direction keys on the keyboard. With the keys Up and Down you select whether to represent a broader or narrower group of related objects. With keys Left and right you can move between groups of objects. Try also to combine the left and right direction keys with the Shift key (see Figure 22).

4.2.7 Query objects

The list of all properties found is sometimes very long (e.g., several hundred). An experienced (advanced) user focuses only on certain properties and does not waste time with others. Sometimes you want to focus only on relations that involve a specific object, for example: *"I think that the triangle ABK is important. I wonder if it is congruent, similar or otherwise related to another triangle."*

Objects in a configuration can be queried if the observed properties are listed in the left pane. The query buttons are located on the taskbar on the right side. To query an object (e.g., a line segment), click on the magenta coloured button for querying the appropriate type of objects and then click on the object (or to the points defining the object). On the properties pane you will then see only properties and relations involving the queried object.

Above the list of properties you can see small buttons with 1-4 dots for controlling the strictness of the query. Less dots means less (and more essential) displayed relations.

Note. A much more powerful query operation is available in OK Geometry Plus.

4.3 Documenting observations. Managing projects

4.3.1 Adding new displayed objects to the construction

Sometimes we want to include objects on a visualised relation in the construction under consideration. For example, OK Geometry detects two congruent triangles (Figure 43). To include these two triangles in the construction, we can proceed in two ways:

- If we press the red **+** button above the visualisation pane (see Figure 22), the added objects will be displayed as emphasized objects.
- If we choose the **+** command, the added objects will be displayed as ordinary objects (Figure 43).

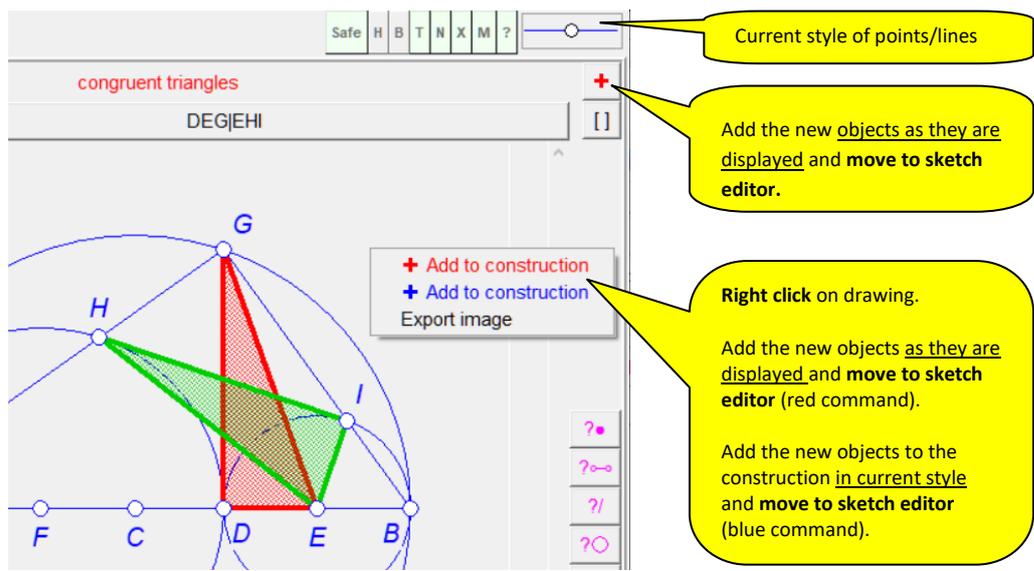


Figure 43

Adding of objects to construction one way or another brings us to the Sketch Editor. In the Sketch Editor, it is sometimes necessary to make some adjustments to the added and emphasized objects:

- Use the **Anchor** button  (possibly together with the mouse wheel) to *position/select an angle between lines* and to move around the point labels.
- Use the **Colour/Style** button  to change the appearance of objects.
- Use the **Hide** button  or **Delete** button  to hide or delete objects.

Figure 44 shows an example of adding object from a displayed relation¹⁰. The detected relation (left) was added to the construction using the + button (centre). The representation was then modified (note the position of angles and labels) with simple editing commands Anchor, Delete, Change colour/style..

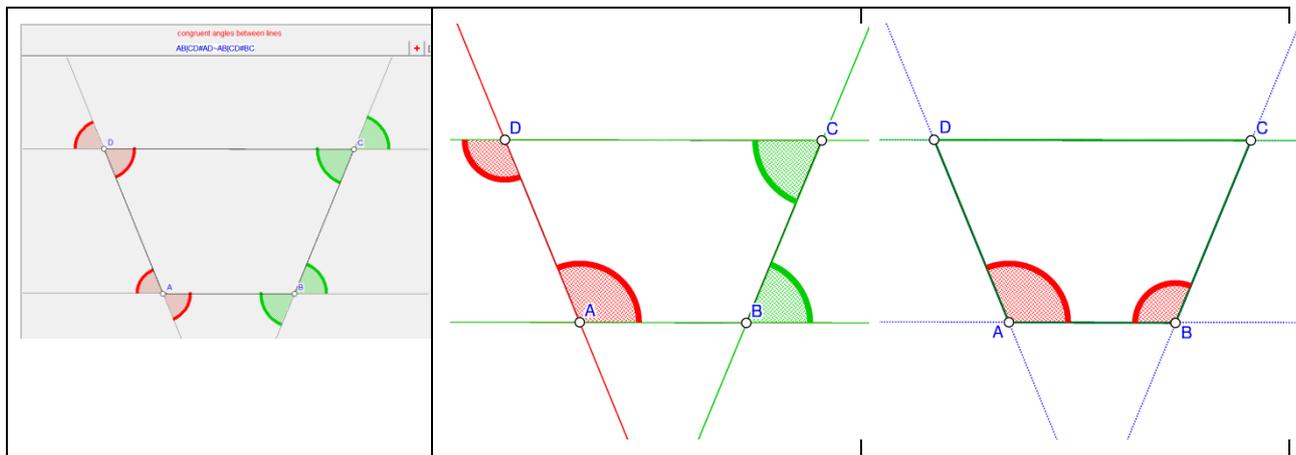


Figure 44

¹⁰ OKExamples\OKG_Basic\Basic_05.p

4.3.2 Declaring a construction as an icon

A construction can be included as an icon into the (current) project. The first icon of the project is called 'Task' by default. To iconise a construction:

1. Declare the current construction as an icon. To do this, click on the button  above the visualisation pane. You can declare the current construction as a **New** icon or overwrite an existing icon.
2. At this stage (or later) it is also possible to comment the icon. See Section 2.2 (Figure 21, Figure 24) for an illustration of this process.

You can use the file *Properties_example.p* in the directory *OKExamples\OKG_Basic* to try the abovementioned process.

4.3.3 Retrieving construction from icons

A project consists of several icons. Each icon contains a construction and a comment, both can be accessed with the  button above the visualisation pane.

4.3.4 Managing the project

A project consists of several icons. Icons can be selected and included into a project for various purposes. Here are some ideas:

- Icons are about various constructions related to a considered geometry problem;
- icons serve as a memo, e.g. a list of interesting properties of a studied construction;
- icons illustrate claims (properties); by dragging and commenting icons one organises the claims into a deductive structure (a proof);
- icons contain tasks for the students.

To visualise all icons of the current project, click the **Project** button in the main menu bar. On the left side you will see the list of icon titles, on the right side you will see the graphical representations of the icons arranged in some way (Figure 45)¹¹. These graphical representations can be dragged around.

¹¹ OKExamples\OKG_Basic\Trapezium2.pro

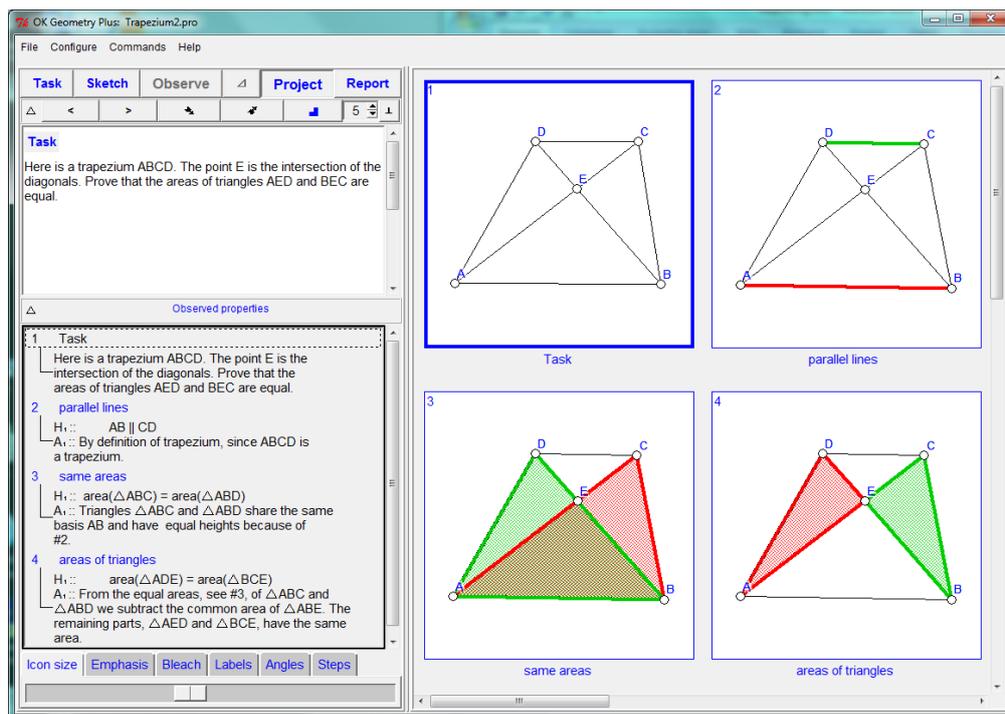


Figure 45

Each icon consists of :

- **The underlying construction.** The underlying construction of the icon can be accessed from the context menu (right click on the icon and apply the command **Edit dynamic construction**). In this way it is possible to work simultaneously with several constructions.
- **The enumeration.** If the icons are organised in a linear structure, they are numbered with the numbers 1,2,3,... In this case, all icons belong to level one. Icons can be demoted to a lower level or promoted to a higher level. A demoted icon becomes a 'sub-icon' of the previous icon of the higher level. For example, icon no. 2 can have sub-icons 2.1, 2.2, 2.3. The icon 2.1 can contain sub-icons 2.1.1, 2.1.2, etc. In this way the icons can be organised in a tree-like structure.
- **Other information.** Each icon has a title and an optional textual comment.

4.3.5 Organising and commenting icons

Icons can be dragged around. Below is the list of operations on icons that are accessible via the context menu (right click on the icon). You can use the file *Properties_example.pro* in the directory *OKExamples\OKG_Basic* to try them.

Command in the context menu	Description
Edit comment	<p>The command is accessible also with a double click the active icon. In the form that appears you can edit the comment as well as the title of the icon.</p> <p>If the comment is a deductive argumentation, it is possible to specify the status of paragraphs (with a right click on paragraphs). A paragraph can be given the status of a</p> <ul style="list-style-type: none"> • definition/assumption/declaration • claim/hypothesis • argument/reason of truth <p>Furthermore, a paragraph can also be a comment or a formula.</p> <p>See Figure 46 for an example of such comment.</p> <p>Simple mathematical expressions can be written using symbols available with  button. Complex formulae and images in general can be written in other software and copy/pasted to OK Geometry – use Image Paste image command in context menu (obtained with a right click).</p>
Edit dynamic construction	<p>This command sets the icon's underlying construction as the current construction in the Sketch Editor. The edited construction can be iconised as a new icon or as an existing icon.</p>
Set icon number	<p>In the form that appears set the new position of the active icon. The new position can be on the same level (<i>after</i> an icon) or on the lower level (<i>right after</i> an icon). For example, assume the current icons are 1, 2, 2.1, 2.2. The position <i>after</i> 2 is 3, while the position <i>right after</i> 2 is 2.3.</p> <p>Note that the icon number 1 (Task) cannot be renumbered.</p> <p>You can increase/decrease the number of the active icon on top level using the + and – keys.</p>
Expand/shrink icon	<p>You can switch between showing the whole collection of icons and showing the icons individually. To switch between the two modes, you can also use the  button below the main menu.</p> <p>Displaying the icons individually is intended for presentations (see 4.3.6).</p>
Promote or demote icon	<p>Icons on level 1 are numbered as: 1, 2, 3 ... Those on level 2 are numbered as 1.1 or 2.4, and those on level 3 like 2.1.4.</p> <p>Demoting an icon increases its level, promoting decreases the level. You can promote (demote) the active icon also by pressing F3 (F4).</p>
Add a copy of icon	<p>The command adds to the project a copy of the highlighted icon.</p>

Delete icon	This command deletes the highlighted icon from the project.
Undo/Redo last	With this command you can undo and redo the last change to the project.
Arrange icons	Icons can be arranged on the pane in various ways: <ul style="list-style-type: none"> - in the form of an Array, - in the form of a Cascade, - in a Free form, as they are positioned by the user, - in the form of a Diagram (Flowchart). <p>The default form is Array.</p>
Edit diagram	The command opens a form for simple editing operations of diagram (flowchart).
Conform icons	Transfer selected properties of an icon to selected icons. With this command you can <ul style="list-style-type: none"> - resize the displayed world area of specified icons so that it matches the displayed world area of a selected icon; - impose the position objects specified icons so that they match the positions of the corresponding objects in a selected icon; - redraw specified icons in accordance to the current drawing parameters (e.g., hatching shapes, drawing angles).
Delete icons ...	In the form that appears just select the icons to be deleted.
Copy / paste icons	In the form that appears just select the icons to be copied. Icons can be pasted to the same project or to another project.
(Modify the appearance of the icons)	Use the sliders at the bottom of the left pane to modify the size of all icons, labels or angle notation, visual emphasis of objects, and the bleaching effect.
(Set the displayed level of icons)	The display level is set on the navigation bar just below the main menu bar (see Figure 26). If the displayed detail-level is set to 2, only icons of level 1 and 2 are be displayed.

4.3.6 Presenting the icons individually

The icons can be shown in project view individually, i.e. one after another, in the same order as in the list in the left pane. First, specify the maximal level of icons to be presented (in case the icons are organised in levels). Choose level 1 to display only the basic points of the work, and level 3 to display all details. To start the presentation, press the  button below the main menu. A big icon will appear

(Figure 46). To switch between icons, use the < and > buttons . Use the >> and << buttons to skip the details (i.e. icons of lower level). Alternatively, use the **PgUp**, **PgDn**, **Home**, **End** keys on the keyboard. During the presentation, you can reduce or enlarge the Task and Properties section in the left pane. To end the presentation, click again the  button.

If the construction contains **scenes**, you can switch between scenes with clicks on scene numbers on the upper right corner of the display (Figure 46). An icon can have several scenes that differ among them only in the emphasis, bleaching or transparent attribute of the objects of the construction in the icon.

In the project view the emphasis and bleaching level are controlled with the sliders in the bottom of the left pane (Figure 46).

You can use the file *Proof_example.pro* in the directory *OKExamples\OKG_Basic* for an exemplary (individual) presentation of icons.

You can use the file *Scenes_example1.pro* and *Scenes_example2.pro* in the directory *OKExamples\OKG_Basic* for an exemplary (individual) presentation of scenes in icons.

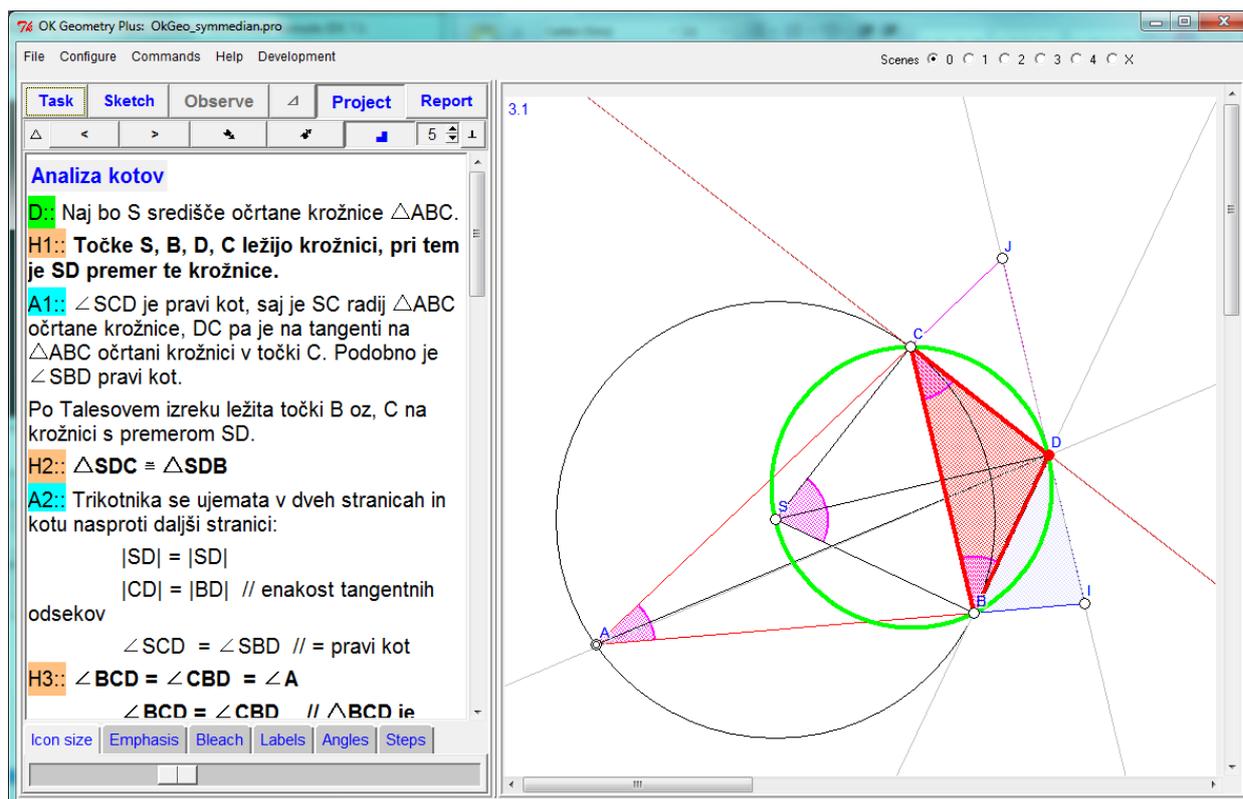


Figure 46

4.3.7 Making a report

A report collects some or all icons of a project, together with their comments, in a document that can be further edited with a suitable application. Note that the Inkscape software should be installed on the

computer if you choose to include in your report images in PDF, EMF, EPSI format or if the images contain symbols.¹²

OK Geometry creates documents in (essentially) three formats:

HTML format (file extensions .htm, .html, .mhtml, .mhtml) is best viewed on computer screen. Preferred format for figures are PNG and SVG.

DOC format (file extensions .doc, .docx) is a standard format for word processors, such as Microsoft Word. Preferred format for figures are PNG and EMF.

TEX format (file extension .tex) is a format used in typesetting applications.

HTML and DOC formats allow several report styles. Icons and comments can be arranged in various ways in columns or otherwise (Figure 48).

Here are the steps for generating a report:

1. Activate the **Report** command (in the Project view).
2. Choose the name of the file that will contain the report. **The format of the file depends on the file extension of your file.**
3. In the form that appears, select the icons to be included in the report.
4. Click on the **Advanced** button and specify the features of the report.
5. Click **OK** to create the report.
6. Probably some features in the obtained report will not be as desired (Figure 47): point labels will be perhaps too small, lines will be perhaps too thin, etc.
7. Repeat the whole process, but specify the options (**Advanced** button) accordingly in step 6.
8. Finish the report in a suitable application.

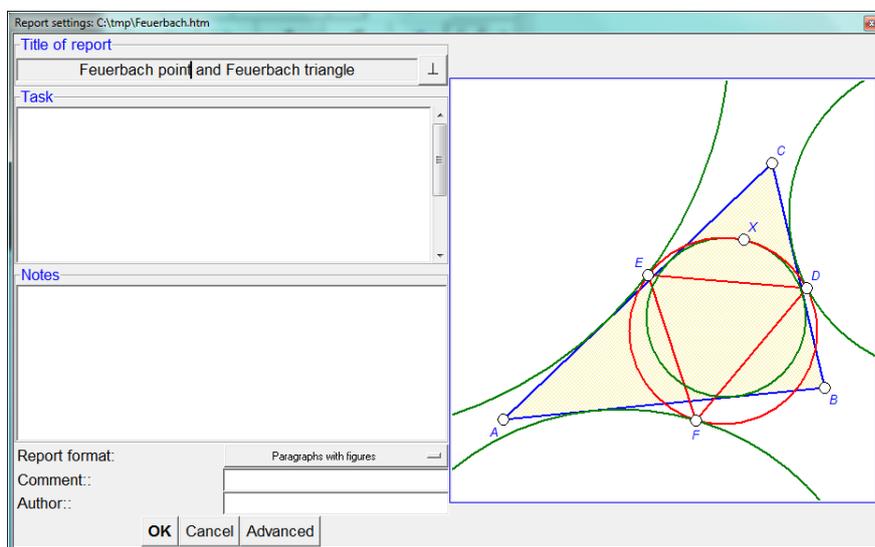
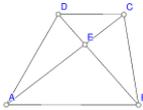
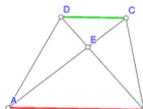
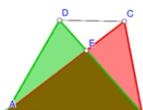
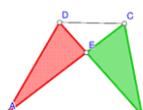
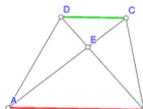
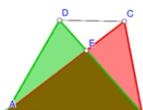
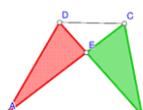
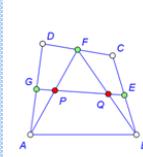
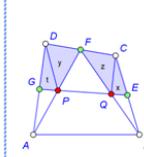
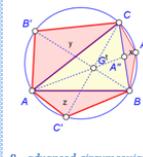
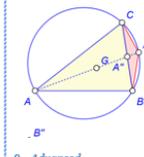
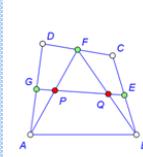
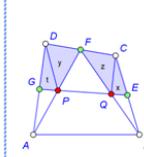
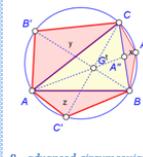
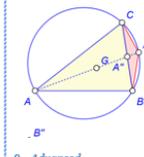
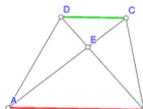
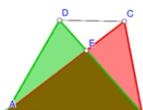
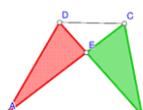
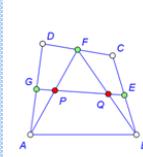
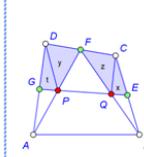
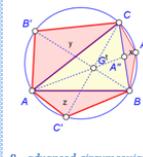
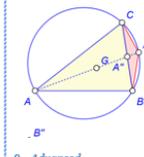


Figure 47

¹² Inkscape is a free software under GPL license. It is available at <https://inkscape.org/>.

Figure 48 illustrates some available styles of the report.

<div style="display: flex; justify-content: space-between; align-items: flex-start;"> <div style="text-align: center;">  </div> <div style="text-align: right;"> <p>Task</p> <p>1 Task</p> <p>Here is a trapezium ABCD. The point E is the intersection of the diagonals. Prove that the areas of triangles AED and BEC are equal.</p> <p>Comment: Two columns with icons</p> </div> </div> <table border="1" style="width: 100%; border-collapse: collapse; margin-top: 10px;"> <tr> <td style="width: 20%; text-align: center;">  </td> <td style="width: 30%;"> <p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p> </td> <td style="width: 50%;"> <p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p> </td> </tr> <tr> <td style="text-align: center;">  </td> <td> <p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p> </td> <td> <p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p> </td> </tr> <tr> <td style="text-align: center;">  </td> <td> <p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p> </td> <td> <p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p> </td> </tr> </table>		<p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p>	<p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p>		<p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p>	<p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p>		<p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p>	<p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p>	<div style="text-align: right; font-size: small; margin-bottom: 5px;"> OK Geometry Basic : 0 - 2022.1.27 </div> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr> <td style="width: 50%; text-align: center;">  <p style="font-size: x-small; margin-top: 5px;">6 <i>advanced_midpoints_1</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p> </td> <td style="width: 50%; text-align: center;">  <p style="font-size: x-small; margin-top: 5px;">7 <i>advanced_midpoints_2</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p> </td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr> <td style="width: 50%; text-align: center;">  <p style="font-size: x-small; margin-top: 5px;">8 <i>advanced_circumcevian</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p> </td> <td style="width: 50%; text-align: center;">  <p style="font-size: x-small; margin-top: 5px;">9 <i>Advanced_2</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p> </td> </tr> </table> <p style="font-size: x-small; margin-bottom: 5px;">Notes</p> <p style="font-size: x-small; color: red;">Observed formula (maxDeg=3, error=2.3e-012, badCases: 0) for d(AA')_{a,b,c,r}</p> <p style="font-size: x-small; color: red;">$*d(AA')^2 * a^3 - 1.4 * a^2 * b^3 - 3.4 * a * b^2 * c^2 + 1.2 * a^2 * b * c^2 - 3.4 * a * b * c^2 = 1.2 * b^3 * c - 1.2 * b^2 * c^2 - 1.2 * b * c^3 = 0$</p> <p style="font-size: x-small; color: red;">Observed formula (maxDeg=5, error=1.1e-014, badCases: 0) for d(DE)_{a,b,c,r}</p> <p style="font-size: x-small; color: red;">$*d(DE) = cA - cB - cC - r = 0$</p> <p style="font-size: x-small; color: red;">Observed formula (maxDeg=3, error=1.1e-014, badCases: 0) for d(DE)_{a,b,c,r}</p> <p style="font-size: x-small; color: red;">$(-1) * d(DE) - cA + cB - cC + r = 0$</p> <p style="font-size: x-small; color: red;">Observed formula (maxDeg=2, error=5.3e-015, badCases: 0) for d(DE)_{a,b,c,r}</p>	 <p style="font-size: x-small; margin-top: 5px;">6 <i>advanced_midpoints_1</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p>	 <p style="font-size: x-small; margin-top: 5px;">7 <i>advanced_midpoints_2</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p>	 <p style="font-size: x-small; margin-top: 5px;">8 <i>advanced_circumcevian</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p>	 <p style="font-size: x-small; margin-top: 5px;">9 <i>Advanced_2</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p>	<p>Two columns with figures</p>	<p>Paragraphs with figures</p>
	<p>2 parallel lines</p> <p>Claim 1 $AB \parallel CD$</p>	<p>Argument 1 By definition of trapezium, since ABCD is a trapezium.</p>														
	<p>3 same areas</p> <p>Claim 1 $\text{area}(\triangle ABC) = \text{area}(\triangle ABD)$</p>	<p>Argument 1 Triangles $\triangle ABC$ and $\triangle ABD$ share the same basis AB and have equal heights because of #2.</p>														
	<p>4 areas of triangles</p> <p>Claim 1 $\text{area}(\triangle ADE) = \text{area}(\triangle BCE)$</p>	<p>Argument 1 From the equal areas, see #3, of $\triangle ABC$ and $\triangle ABD$ we subtract the common area of $\triangle ABE$. The remaining parts, $\triangle AED$ and $\triangle BCE$, have the same area.</p>														
 <p style="font-size: x-small; margin-top: 5px;">6 <i>advanced_midpoints_1</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p>	 <p style="font-size: x-small; margin-top: 5px;">7 <i>advanced_midpoints_2</i></p> <p style="font-size: x-small;">In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p>															
 <p style="font-size: x-small; margin-top: 5px;">8 <i>advanced_circumcevian</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p>	 <p style="font-size: x-small; margin-top: 5px;">9 <i>Advanced_2</i></p> <p style="font-size: x-small;">G is the centroid of ABC.</p>															

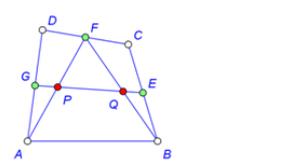
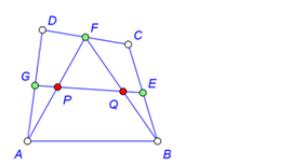
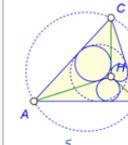
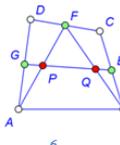
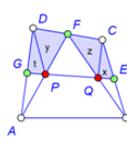
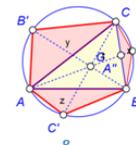
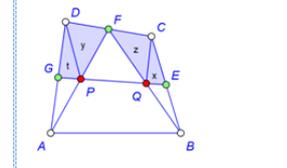
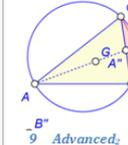
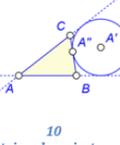
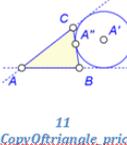
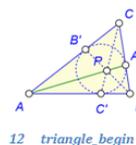
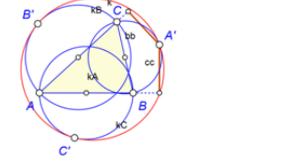
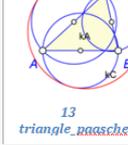
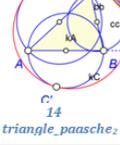
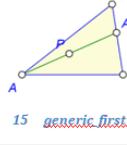
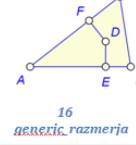
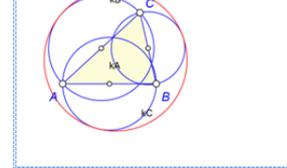
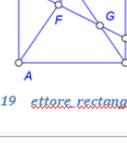
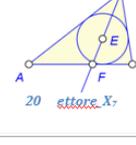
<p>6 <i>advanced_midpoints₁</i></p> <p>In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p> 		 <p>5 <i>simple_dodatni_circles</i></p>	 <p>6 <i>advanced_midpoint_{s1}</i></p>	 <p>7 <i>advanced_midpoints₂</i></p>	 <p>8 <i>advanced_circumscribed_xian</i></p>
	<p>7 <i>advanced_midpoints₂</i></p> <p>In a convex quadrilateral ABCD let E, F, and G be the midpoints of sides BC, CD, and DA. Furthermore, let P and Q be the intersections of line GE with lines AF and BG.</p>	 <p>9 <i>Advanced₂</i></p>	 <p>10 <i>triangle_pictura</i></p>	 <p>11 <i>ConvOptimalna_pisat_ana</i></p>	 <p>12 <i>triangle_begin</i></p>
<p>14 <i>triangle_pasche₁</i></p> <p>r = radius of the circle k bb = distance from A' to line AC cc = distance from A' to line AB</p>		 <p>13 <i>triangle_pasche</i></p>	 <p>14 <i>triangle_pasche₂</i></p>	 <p>15 <i>generic_firxi</i></p>	 <p>16 <i>generic_razmerja</i></p>
	<p>13 <i>triangle_pasche</i></p> <p>r = radius of the circle k.</p>	 <p>17 <i>ettorx_nagel</i></p>	 <p>18 <i>ettorx_paralelogram</i></p>	 <p>19 <i>ettorx_rectangle</i></p>	 <p>20 <i>ettorx_X₇</i></p>

Figure 48

4.4 Menu commands

Here is a brief overview of the available menu commands:

4.4.1 File Commands

Command	Meaning	Comments
New project	Begin a new project	Begin a new project. Deletes all icons as well as the current construction and starts the Sketch Editor.
Open Open construction	Open (import) an OK Geometry construction file Import a construction from Cabri, Cabri Express, GeoGebra, Cinderella, C.a.R. (Compass and Ruler), Sketchometry, JGEX or OK Geometry file	Read a construction that was saved in OK Geometry or in some other format. The read construction overrides the current construction.
Open Open project	Open a project	Reads a project that was previously saved to a file with .pro extension.
Save Save construction	Save the current construction to a file	The current construction is saved as OK Geometry file with .p extension.
Save Save project	Save the current project	The current project is saved as OK Geometry file with .pro extension.
Save Save	Save the current work to the related .p or .pro file.	
Export Export image	Export image to clipboard, raster file or vectorised file	
Export Export to static GeoGebra construction	Export image to static GeoGebra construction	
Recover last session	Recover last session.	OK Geometry regularly stores the current work. In case of some kind of failure with this command the session can be recovered immediately after the start of a new session.

Inspect files	Overview dynamic constructions in various formats	Inspection of files in various dynamic geometry formats with the visualisation of the related constructions.
Exit	Exit from OK Geometry	

4.4.2 Configuration Commands

Command	Meaning	Comments
Working mode	Choose between the Easy mode, the Basic mode, and the Plus mode of work using OK Geometry	<p>The Easy mode is intended for occasional users. In this mode it is possible only to analyse imported geometric constructions or produce reports (e.g. proofs) from previously schemas. See Help for Simple mode for details.</p> <p>The Basic mode is intended for users who want to have full control over OK Geometry when studying geometric situations. The Basic mode encompasses a powerful geometry editor (see Sketch Editor Reference).</p> <p>The Plus mode is intended for advanced users, interested in triangle geometry, construction problems, creating geometry tasks, etc.</p>
General options	Set the general configuration data	Here it is possible to set several working options (language, size of fonts, etc.), as well as specific options for the Sketch Editor, observation parameters, project operations, and project report.
Considered properties	Information about the properties that are checked for various levels of analysis.	The command displays the considered properties in observation for various levels of analysis.
Load configuration	Load a previously saved configuration of OK Geometry	A configuration of OK Geometry can be stored and retrieved.
Save configuration	Save the current configuration of OK Geometry	A configuration of OK Geometry can be stored and retrieved.

4.4.3 Workflow Commands

This menu contains mainly commands from the main menu bar and some special commands available only in the Plus mode.

Command	Meaning	Comments
Slightly shake the construction	Slight modification of the current construction (F6)	The free points in the current dynamic construction are slightly moved. The movements are minor so that the construction remains essentially the same.
Construction steps	Display construction steps and scenes of the current construction	

4.4.4 Help Commands

The meaning of menu entries is self-evident.

4.4.5 Shortcuts

Shortcuts in construction view

Shortcut	Meaning
Escape	Stop the execution of the command (where applicable)
F1	Help (Find command) Glossary (Plus mode)
F2	Save the current file
F5	Table with characters
F6	Shake points (only in perturbed view)
F7	Switch between perturbed and exact view
F8	Graphic help on/off
Ctrl + Z or Ctrl + Y Ctrl + R or Alt + BackSpace	Undo / Redo construction step
Ctrl + Mouse wheel Ctrl + Plus/Minus	Increase/decrease zoom level
Ctrl + Home	Next (cyclic) zoom
Shift + Mouse wheel Shift + Up/Down	Change degree of emphasis of objects
Alt + Mouse wheel	Change degree of bleaching of objects

Alt + Up/Down		
Alt + F	Anchor (modify label position, angle selection, congruence sign, area fill, arrow type) for objects	
Alt + S	Set label for Auto label	
Alt + A	Auto label	
Alt + B	Label all	Only in Sketch Editor
Alt + S	Set label for Auto label	
Alt + P	Point	
Alt + L	Line 2 objects	
Alt + C	Circle: center + point	
Alt + N	Enter a number or parameter	
Alt + H	Hide object	

Shortcuts in project view

Shortcut	Meaning
Escape	Stop the execution of the command (where applicable)
F2	Save the current file (construction or project)
F3	Promote icon
F4	Demote icon
Ctrl + Z	Undo last operation on icons