# OK Geometry

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## 1   What is OK Geometry ?

OK Geometry is a tool for analysing geometric constructions. It detects patterns and geometric and algebraic properties of construction. You can think of OK Geometry as a pair of geometric spectacles for observing invariants of dynamic geometric constructions. OK Geometry can also help to prove and to organise the information found.

OK Geometry can help students to notice properties of constructions that they may not have been aware of. Observing and conjecturing properties of geometric objects and configurations are important processes, especially when proving facts. However, students need to be aware that it is one thing **to observe** something, another thing is **to know that** something is true, and yet another thing is **to know why** something is true. OK Geometry helps with observation and can help to find out and understand why something is true. You can also ask OK Geometry to prove facts.
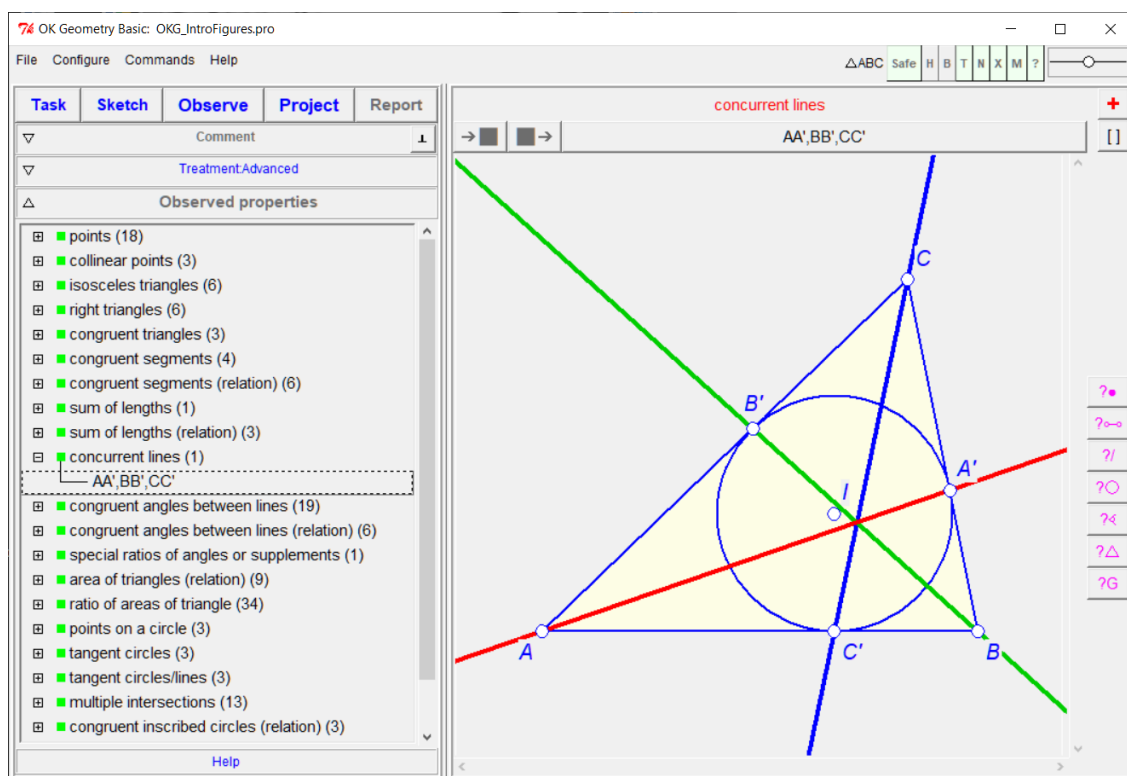


OK Geometry operates in three modes. The **Easy mode** is intended for occasional users and for students learning the concept of deductive proof in the geometry context. The **Basic mode** is intended for teachers and students who like to study challenging problems in planar geometry. The **Plus mode** is intended for enthusiasts who want to discover and perhaps prove interesting facts in planar geometry, as well as for specialists in the field of triangle geometry. Easy and Basic modes are available in English, Czech, German, Italian and Slovenian language.

# 2   What can OK Geometry do?
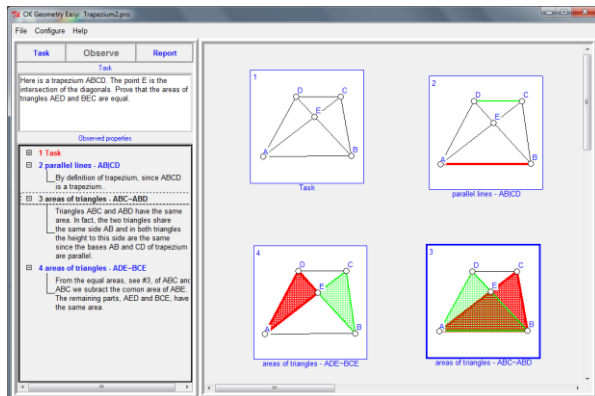


For students learning to prove geometric facts, OK Geometry can be an **environment for solving prepared simple proof tasks.** The prepared problem usually contains selected properties that must be correctly ordered, provided with deductive arguments, and structured into a proof.

For casual users, OK Geometry is **a tool for detecting properties of dynamic constructions** created in the dynamic geometry software they use. OK Geometry can **import dynamic constructions** created with some of the most popular dynamic geometry programs (Cabri Geometre, Cabri Express, GeoGebra, Cinderella, Z.u.L. (Zirkel und Linien), JGEX, Sketchometry ).

For example, in the construction on the right, points A', B', C' are the points of contact of the incircle of the triangle ABC with its sides. The points A'', B'', C'' are the intersections of the incircle with the segments connecting the centre I of the incircle with the vertices of the triangle.  OK Geometry observes that



- A'' is the orthocentre of the triangle AC'B';
- the lines AA', BB', CC' concur in a point;
- the lines A'A'', B'B'', CC'' also concur at a point, say P;
- many many other properties.
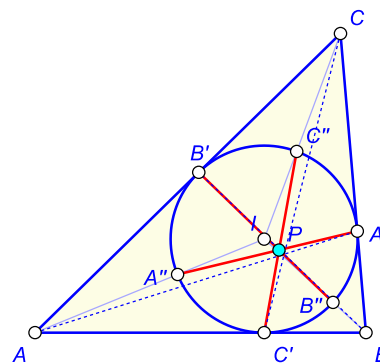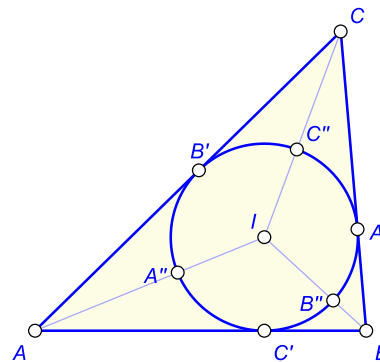
A further **triangle analysis with OK Geometry** of the above example shows that

- the point of concurrence of the lines AA',BB', and CC' is the Gergonne point (X7) of the triangle ABC;
- the point P of concurrence of lines A'A'', B'B'', C'C'' is the so-called mid-arc point (X177) of the triangle ABC;
- P is the centre of the incircle of the triangle A'B'C';
- P lays on the Feuerbach hyperbola of the triangle ABC;
- P lays on the line through the incentre I and the point X167 (the Nagel point of the excentral triangle of ABC),
- And many many other properties.
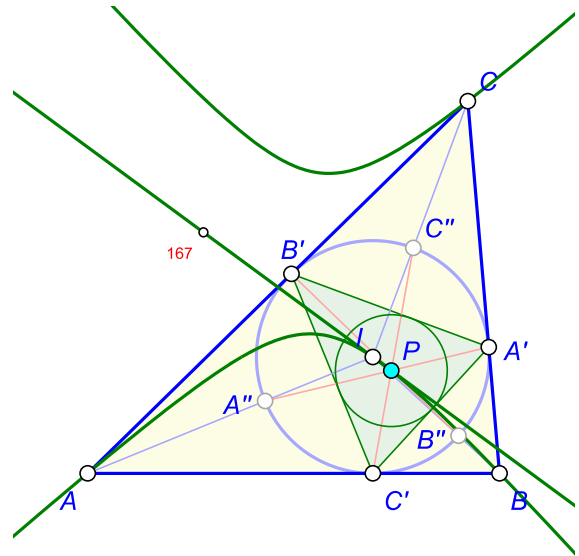
When considering triangles, OK Geometry takes into account a rich database of triangle centres (about 60000), circles, lines, conics, cubics and various triangle transformations.

OK Geometry is equiped with an **extensive glossary of characteristic triangle objects**. Many concepts of triangle geometry are explained, illustrated and referred to related commands.
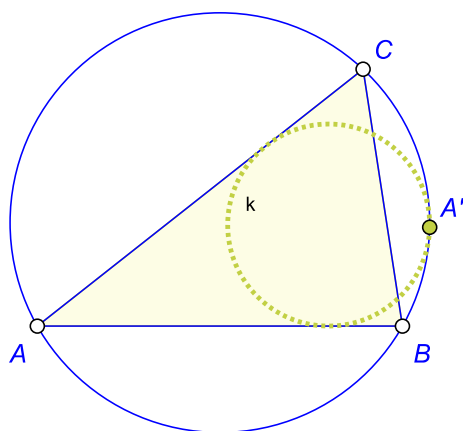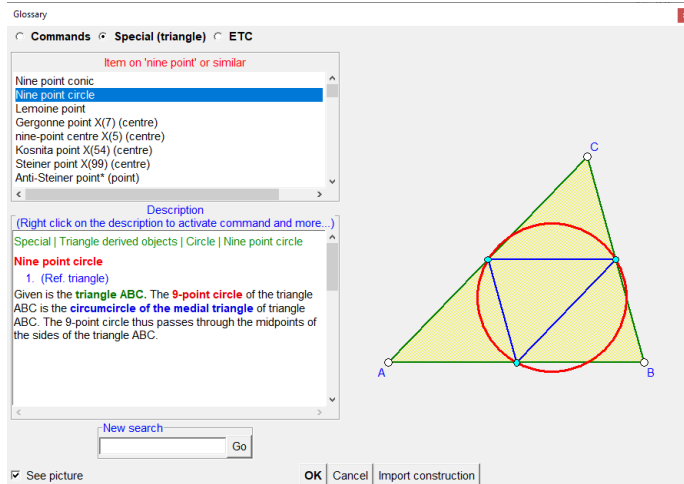
With OK Geometry you can **create objects that are difficult to construct**, such as a circle that is tangent to three other circles, or an object that meets some optimisation criteria.

Suppose we want to construct a circle *k* that touches two sides of a given triangle ABC and its circumcircle. You can do this with a simple command in OK Geometry. Once the circle is created, you can construct the point A' of contact of the two circles. Hopefully, by examining the properties of the point A', you find a way to properly construct the circle *k* (using compass and ruler or some dynamic geometry software).
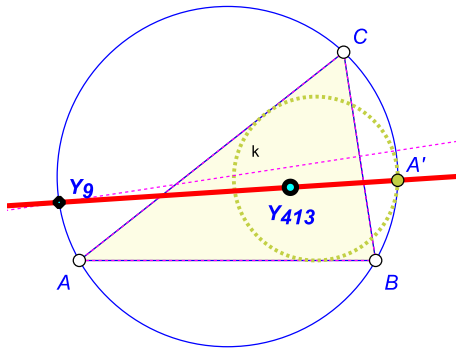
With OK Geometry you can **observe the relationships of an inspected object to objects that are not part of the construction**.

For example, in the above configuration OK Geometry finds that the point A' lays on the line through the points
 Y413 = centre of incircle of triangle ABC, and
 Y9    =intersection of the circumcircle of ABC and the bisector of the side BC.

OK Geometry can **observe algebraic relations** between geometric quantities within a construction.

In the above configuration OK Geometry finds several expressions for the radius *rk* of the circle *k* or for the ratio of distances |A'B| : |A'C|. Here are just two of them:

$$rk = \frac{2 \cdot r \cdot (a + b - c) \cdot (a - b + c)}{a \cdot (a + b + c)}$$

$$\frac{|A'B|}{|A'C|} = \frac{(a - b + c)/b}{(a + b - c)/c}$$

In the expressions, *a, b, c* are the side lengths of the triangle, and *r* is the radius of its circumcircle.
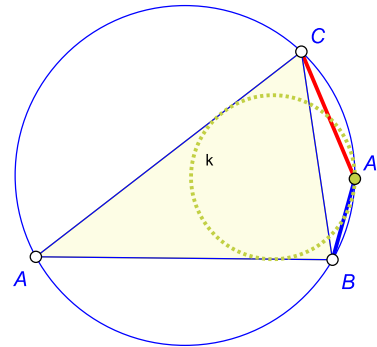
OK Geometry is primarily intended for the observation of facts in plane geometry and the discovery of conjectures. However,  since proofs are the essence of mathematics, OK Geometry also **provides deductive proofs of geometric properties**. The prover is based on the Geometric Deductive Database method and, when successful, provides a readable proof that can be visualised to some extent.

The prover is also used to identify properties of configurations that are difficult to prove.
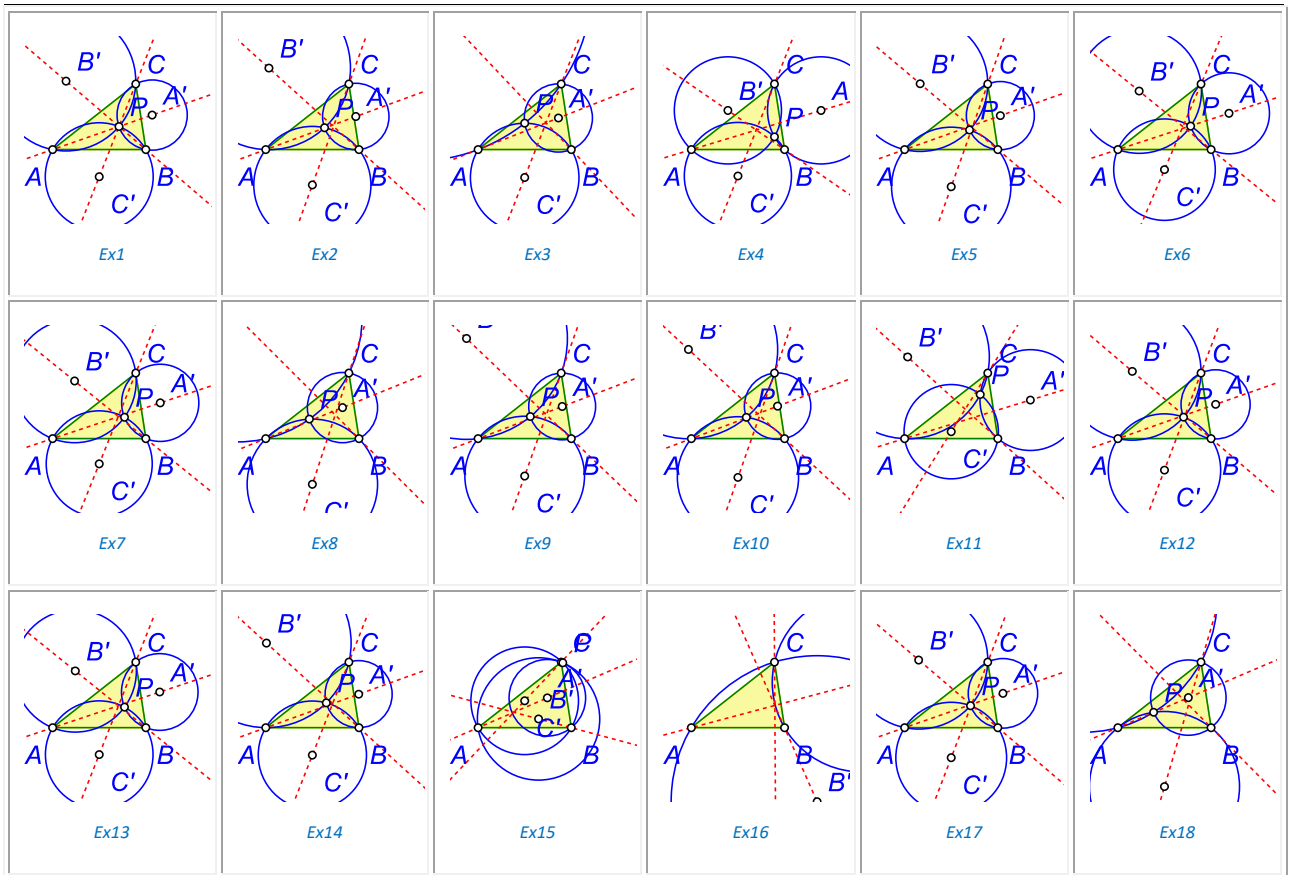
In OK geometry it is also possible to **create families of constructions** and **observe the properties of all examples in the family**. A family can consist of tens, hundreds or more constructions.

For example, consider the construction on the right: let P be the orthocentre of a given triangle ABC and let A' be the centre of the circle through B, C, and P. We define the points B' and C' cyclically.
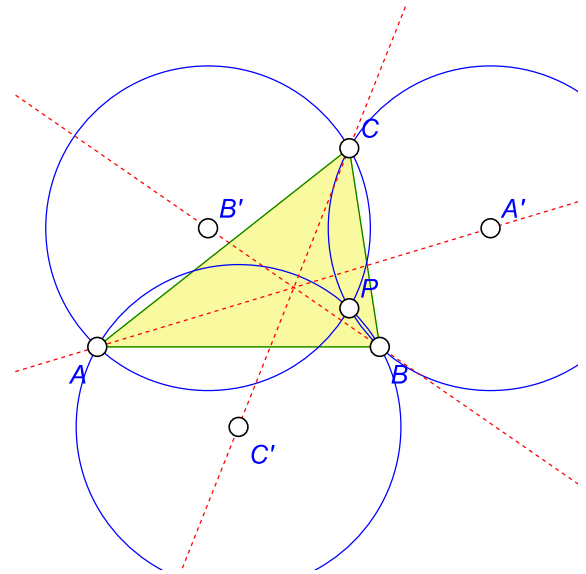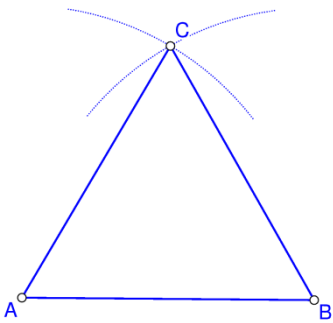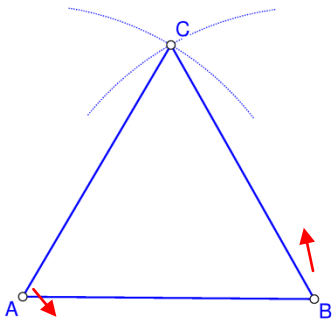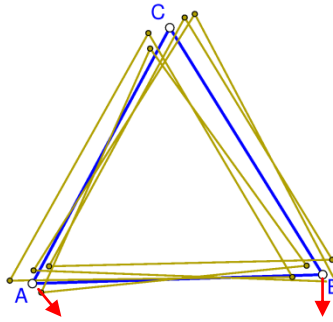
It is known that the lines AA', BB', CC' concur in a point. Do they also concur if P takes the position of other triangle centres?

To investigate this, we create a family of constructions, in which P takes the position of, say, centres X1 through X18. OK Geometry observes that the AA', BB', CC' concur only when P takes the position of X1, X3, X4, X13, X14, X15, and X16. You can also easily obtain the properties of the points of concurrence.



| Ex1 | Ex2 | Ex3 | Ex4 | Ex5 | Ex6 |
| Ex7 | Ex8 | Ex9 | Ex10 | Ex11 | Ex12 |
| Ex13 | Ex14 | Ex15 | Ex16 | Ex17 | Ex18 |

# 3   How does OK Geometry observe properties?

The model behind OK Geometry is the stochastic dynamic model. This model is similar to the dynamic model in that the non-constructed (free) objects can be dragged, while the constructed objects are modified according to the construction procedure. In the stochastic-dynamic model, however, the non-constructed objects are represented by multiple copies of randomly positioned objects. The constructed objects are represented by several objects constructed from copies of other objects in accordance with the construction. Only one copy of each object is visible, and takes a reasonably random position (as well as other copies).

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| The static model. All represented objects are fixed. | The dynamic model. Free (non-constructed) points can be dragged. All other objects change accordingly. | The stochastic dynamic model. Free (non-constructed) points are represented by several copies of randomly positioned points and can be visually dragged. Only one of them is visible. |

Simply put, OK Geometry reads dynamic constructions (not positions). The program then creates multiple copies of the construction by randomly moving all copies of all free points. The properties that are invariant (in terms of measurement) to such perturbations are considered as "observed". But not proven, of course.

# 4   Who is the author of OK Geometry?

The author of OK Geometry is dr. Zlatan Magajna. He is a retired lecturer in didactics of mathematics at the Faculty of Education, University of Ljubljana, Slovenia.

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